The complexity of market dynamics: computational intelligence assisted investigations

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Abstract:

The tangled topic of market dynamics is relatively new problem in economic science. In clear form we meet dynamic models and/or theories even in modern era of economics that is in past hundred years approximately since 1925. In the middle of past century several economists like von Neumann, Kaldor, Samuelson, Goodwin, Smithies, Domar, Metzler, Haavelmo, Klein, Hicks, Lange, Koopmans and others have developed dynamic models concerning the stability and fluctuations around any equilibrium point or path which cover the four important fields of economic theory, namely, business cycles, income determination, economic growth and price theory. On the other hand there are few economists which imagined about economic dynamics before 20th century, at least in hidden form like A. Cournot in his mathematical theory of duopoly. However there exists one economist who was reasoning about dynamics in economic evolution very early. His name is Ibn Khaldun. He focused attention on that important characteristic of economic phenomena, even in the period of so called Golden Arabic medieval, that is, several centuries earlier than economists in modern era.

Because in a market there are a lot of multilevel discrete time action-reactions, it is very difficult to build model and/or theory upon such realms. Even if there are only two levels of iterative reactions that is mutual reaction between supply and demand or between two producers on the supply side of market (duopoly) the model may need very sophisticated mathematics. These are the reasons why mathematically formalized models are becoming too peculiar for conventionally educated economists to create suitable imaginations. In such situation computational intelligence (CI) devices and tools are coming to help for creating clearer imaginations about such complex realms. Actually, computational economies as a manmade object similar to concept of Kripke´s "possible world" have several advantages and benefices in endeavour to create imaginations on economic reality relate to bodies created in mathematical economies.

The typical distinguishing feature in creation economic imagination in the approach assisted by ICT is the direct observation of evolution running in PC or in poorer case the static snapshots from the PC display. In such of paper printed cases the verbal abstract text is substituted by lot of coloured pictures. This needs the readers to learn how to "read" such on paper printed snapshots from PC or in better case of running simulation one.

The author in this essay is bringing approaches to complex dynamics realms of market using relatively simple mathematical models for demonstration economic dynamic complexities mainly by assistance of program iDMC and Excel too. These are two: 1. Duopoly model a lá Cournot, 2. Duopoly model with two mutually different products.

Keywords:

Basin structure, Competent ontology, Computational economics framework, Evolutionary dynamism, Iterative process, Market dynamism, Discrete dynamical systems, Duopoly models, Subcritical Neimark–Hopf bifurcation, Homoclinic connection.

JEL - Journal of Economic Literature classification system:

Microeconomics, Economic Methodology, Current Heterodox Approaches, Economic Systems.

Introduction

"Not only verbal texts but to read snapshots from PC runs:
that is the way to understand economic dynamics.
The best one is direct observation of runs."

There are several strange and complex phenomena and process for recognition economies. Among other the most difficult subject is economic dynamics. But this term is rather vague among economist. So it is legitimate to ask the question: "What is the meaning of Market Dynamics?

The market is complex action-counteraction hidden below price system entity which is evolving in discrete time steps. In such situations it is obviously that process dynamics has an iterative form. Obviously it is modelled as discrete dynamical system (system dynamics[1]), that is a model of rigid body walking[2] step by step upon state and/or phase space (as state and/or phase point). That is the reason why we need to use discrete mathematic tools for example topology. Market dynamics is visualised through pricing signals that are created as a result of changing supply and demand levels in a given market.

[1] As it is known the System dynamics is a methodology and mathematical modelling technique to frame, understand, and discuss complex issues and problems. It was created during the mid-1950s by Professor Jay Forrester of the Massachusetts Institute of Technology. After massive diffusion of ICT devices different dynamical models runs in PC´s.

[2] Because the length of steps are differing the state/phase point is sometimes bounces/dances and/or faltering around.

Market dynamics describes the dynamic, or change of price signals that results from the continual changes in both supply and demand of any particular product or group of products. Market dynamics is a fundamental concept in supply, demand and pricing economic models.

The several topics of market dynamics or at least all of them are relatively new problem area in economic science. In clear form we meet dynamic models and/or theories even in modern era of economic that is in past hundred years approximately since 1925. In the middle of past century several economists like von Neumann, Kaldor, Samuelson, Goodwin, Smithies, Domar, Metzler, Haavelmo, Klein, Hicks, Lange, Koopmans and others have developed dynamic models concerning the stability and fluctuations around any equilibrium point or path which cover the four important fields of economic theory, namely, business cycles, income determination, economic growth and price theory. On the other hand there are few economists which imagined about economic dynamics before 20th century, at least in hidden form like A. Cournot in his mathematical theory of duopoly. However there exists one economist who was reasoning about dynamics in economic evolution very early. His name is Ibn Khaldun. He focused attention on that important characteristic of economic phenomena, even in the period of so called Golden Arabic medieval, that is, several centuries earlier than economists in modern era.

Because in a market there are a lot of multilevel discrete time action-reactions, it is very difficult to build model and/or theory upon such realms. Even if there are only two levels of iterative reactions that is mutual reaction between supply and demand or between two producers on the supply side of market (duopoly) the model may need very sophisticated mathematics. These are the reasons why mathematically formalized models becoming too peculiar for conventionally educated economists to create suitable imaginations. In such situation computational intelligence (CI) devices and tools are coming to help for creating clearer imaginations about such complex realms. Actually, computational economies as a manmade object similar to concept of Kripke´s “possible world” have several advantages and benefices in endeavour to create imaginations on economic reality relate to bodies created in mathematical economics. The typical distinguishing feature in creation economic imagination in the approach assisted by ICT is the direct observation of evolution running in PC or in poorer case the static snapshots from the PC display. In such of paper printed cases the verbal abstract text is substituted by lot of coloured pictures. This needs the readers to learn how to “read” such on paper printed snapshots from PC or in better case of running simulation case.

We are bringing some unconventional approaches to complex dynamics realms of market using relatively simple and in economic literature known mathematical models for demonstration economic dynamics complexities mainly by assistance of program iDMC and Excel too. These ones are two: 1. Duopoly model a lá Cournot, 2. Duopoly model with two mutually different products.

1 Investigation of simple but modified duopoly model a lá Cournot

There are several duopoly models found in economic literature. For better understanding we are chosen for beginning original duopoly model only with modest customization. It consists in introduction of two adaptation parameters *b* and *c* (in Excel model *bas* and *cas*) into 2D difference system:

$$x_{t+1} = (1-c)x_1 + c\left(\sqrt{\frac{y_1}{a}} - y_1\right)$$
$$y_{t+1} = (1-b)y_1 + b\left(\sqrt{x_1} - x_1\right) \qquad (1)$$

where parameter *a* is a multiply of marginal cost of production of first producer when $mcost_y = 1$. Understandably, if $b = c = 1$ the situation is in common sense of original duopoly model, that is:

$$x_{t+1} = \sqrt{\frac{y_1}{a}} - y_1$$
$$y_{t+1} = \sqrt{x_1} - x_1 \qquad (2)$$

The 2D difference system (1) and/or (2) are topological map, of course:

$$T_{Modif} : \begin{cases} x' = (1-c)x + c\left(\sqrt{\frac{y}{a}} - y\right) \\ y' = (1-b)y + b\left(\sqrt{x} - x\right) \end{cases} \quad \text{and/or} \quad T_{Orig} : \begin{cases} x' = \sqrt{\frac{y}{a}} - y \\ y' = \sqrt{x} - x \end{cases} . \qquad (3)$$

To understand the presentation of market dynamics in the form of unconventional exposition we use besides abstract texts the snapshots from PC simulation. We are bringing them for readers to learn several subsequent snapshots.

The first two snapshots are made in iDMC from the original duopoly map (right part of map (3)). We choose two starting points for parameter *a*, that is $a = 5.5$ and $a = 6$ to show attractive and repellent situations, see Figure 1 and Figure 2.

Let us now disturb Cournotian equilibrium by introducing parameter *b* and *c* such that both are coming in interval $0 < (b, c) < 1$, that is we decelerate the reaction of producers to the situation in the market. For example we chose $b = c = 0.85$ and $a = 7.2$ see Figure 3. We encourage the readers for attention to learn how to read subsequence snapshots from Excel and iDMC simulations.

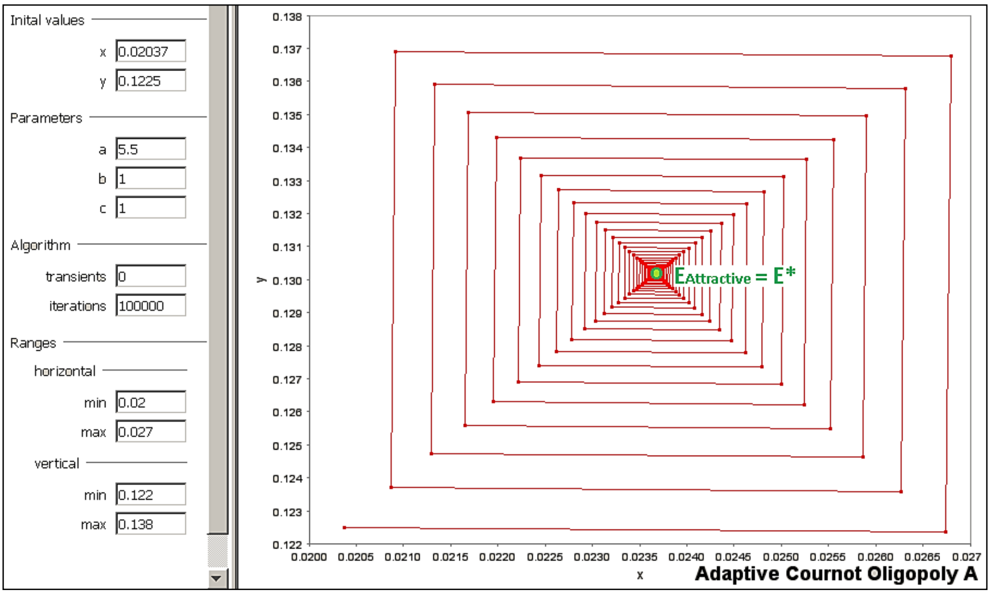

Figure 1. The attractive Cournot point

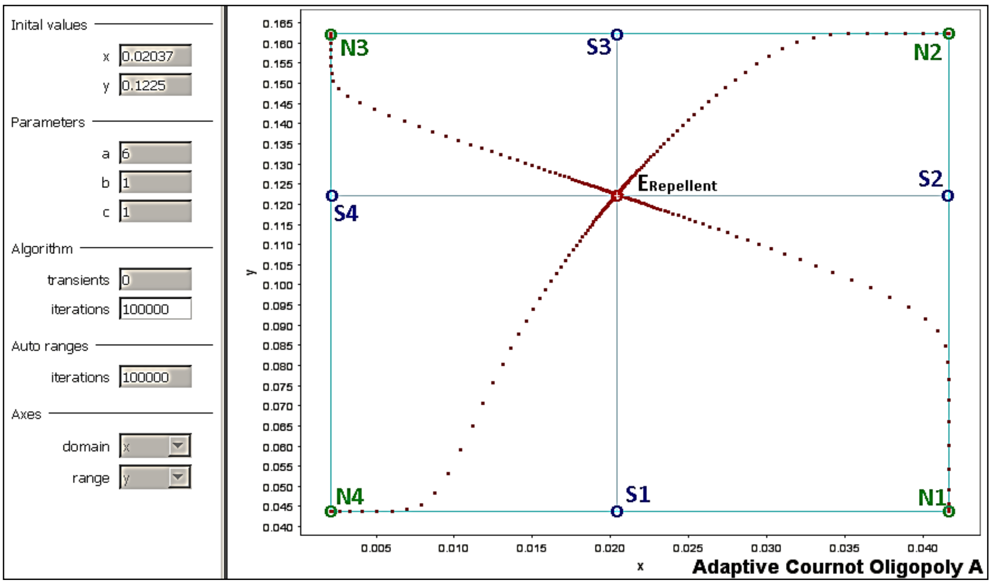

Figure 2. The repellent Cournot point

The result of such learning is very important for progress of theirs capabilities to create genuine economic imaginations and it promise the advance of their adequate economic reasoning. We give to those snapshots only little commentaries to leave broader space for reader's creativity. By the way these are one of the main didactical advances of this unconventional approach.

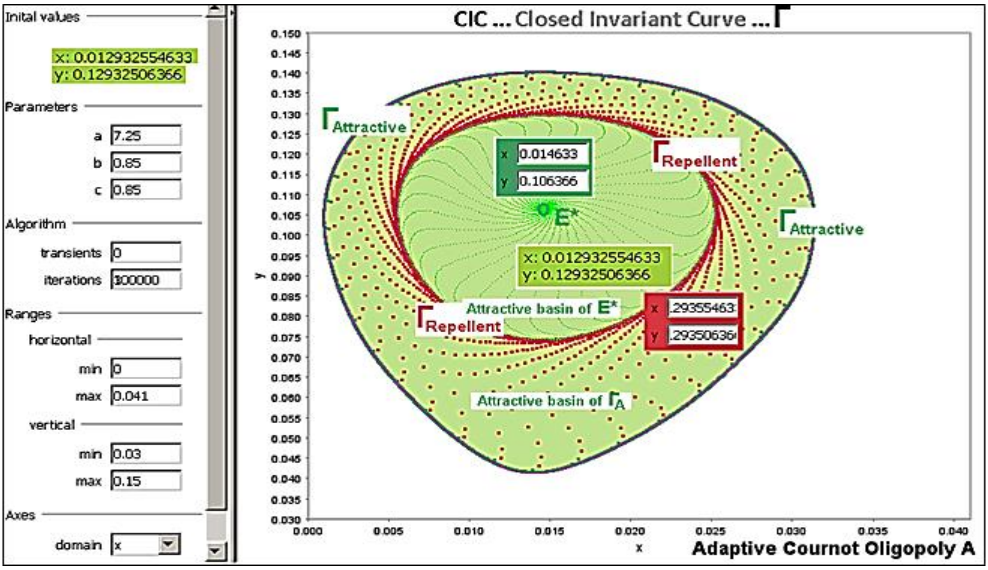

Figure 3. Situation after introduced parameters b and c

The snapshot in Figure 3 we combined from four snapshot reached by different algorithms of iDMC device by superposing them in Windows. The resulting picture shows the coexistence of attractive Cournot point and the outer attractive closed invariant curve (CIC), between them other CIC is laying but repellent.

In Figure 3 we can see also basin of attraction of E* and two basins of attraction of attractive G_A.

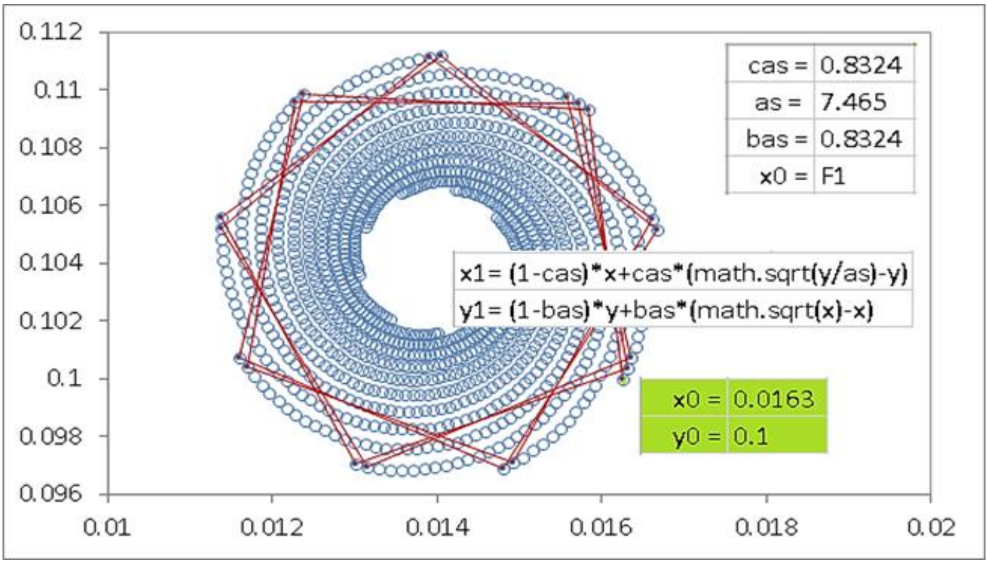

Figure 4. The duopoly: Evolving to Cournot Equilibrium

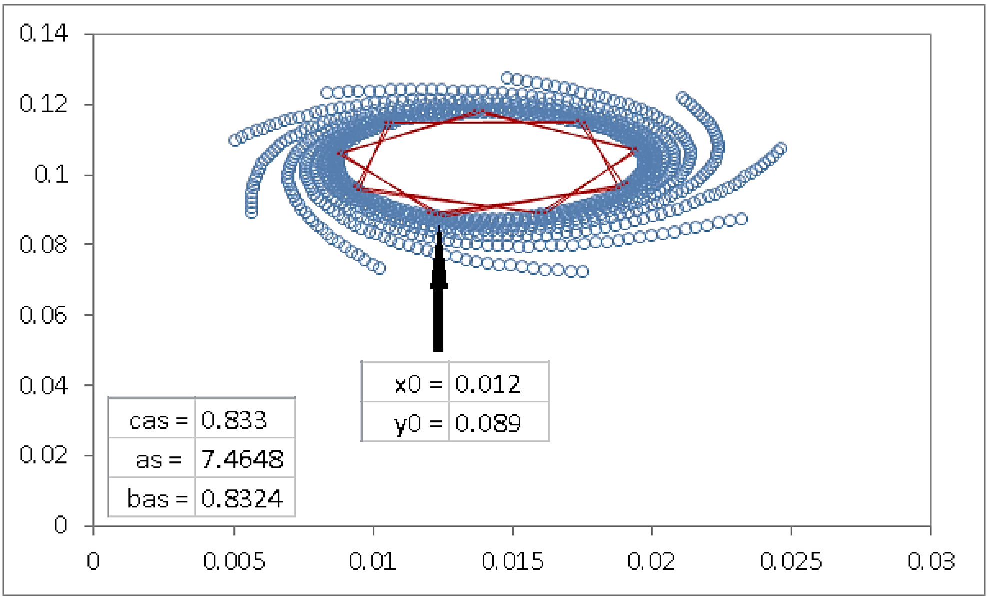

Figure 5a. The unstable behaviour (the Cournot point is repellent)

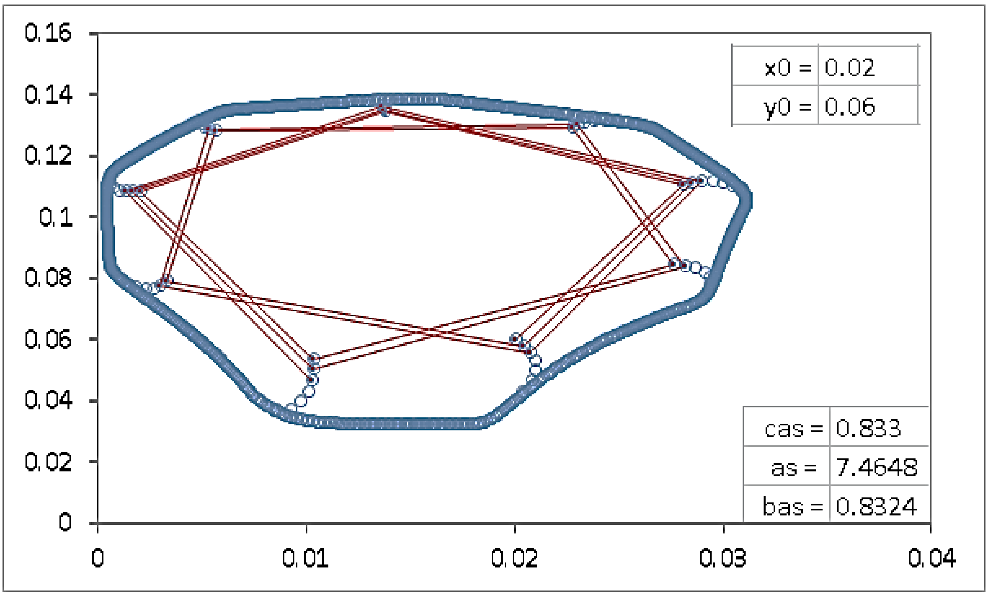

Figure 5b. Longer run show attraction to orbit
(after approaching 9-edge polygon rotates upon it)

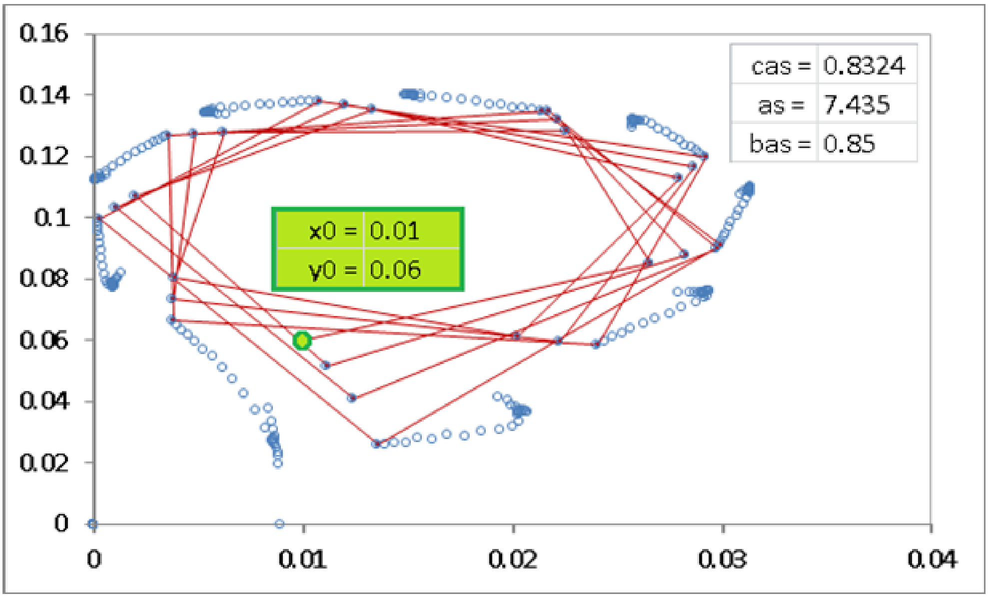

Figure 6. More strange behaviour of duopoly

As readers easy recognise we have used Excel for preparing Figure 4 to Figure 7 knowing that for economists the Excel is most convenient device. Some snapshots from Excel may be more instructive but very complex dynamics is needed to deal with iDMC.

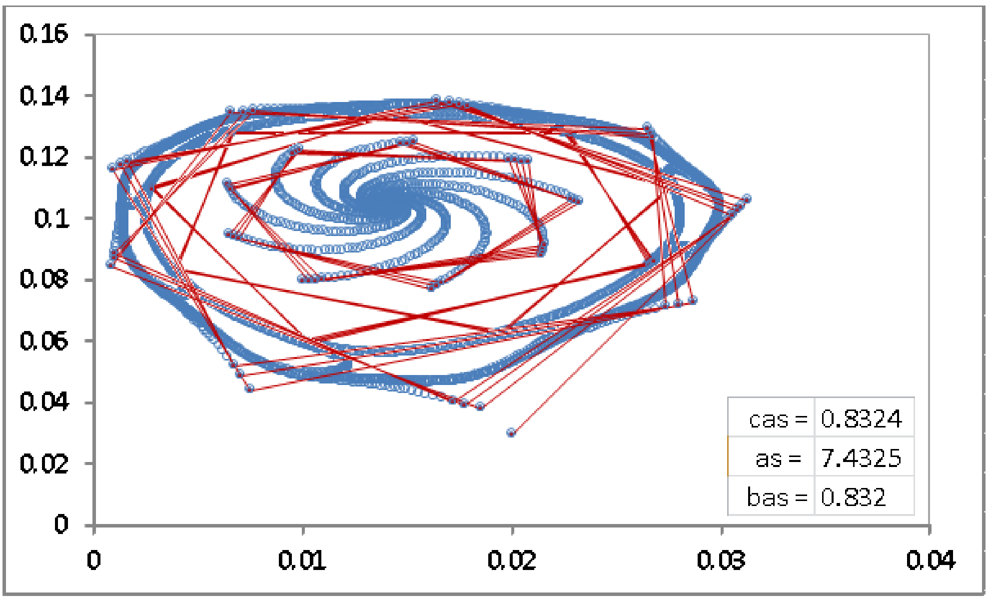

Figure 7. Three runs from different starting points

The reader can assure precisely observing the snapshot in Figure 8 and of subsequent ones. The careful reading of snapshot on Figure 8 opens very deep information on simple market dynamics and prevent readers from naïve imaginations about real economy.

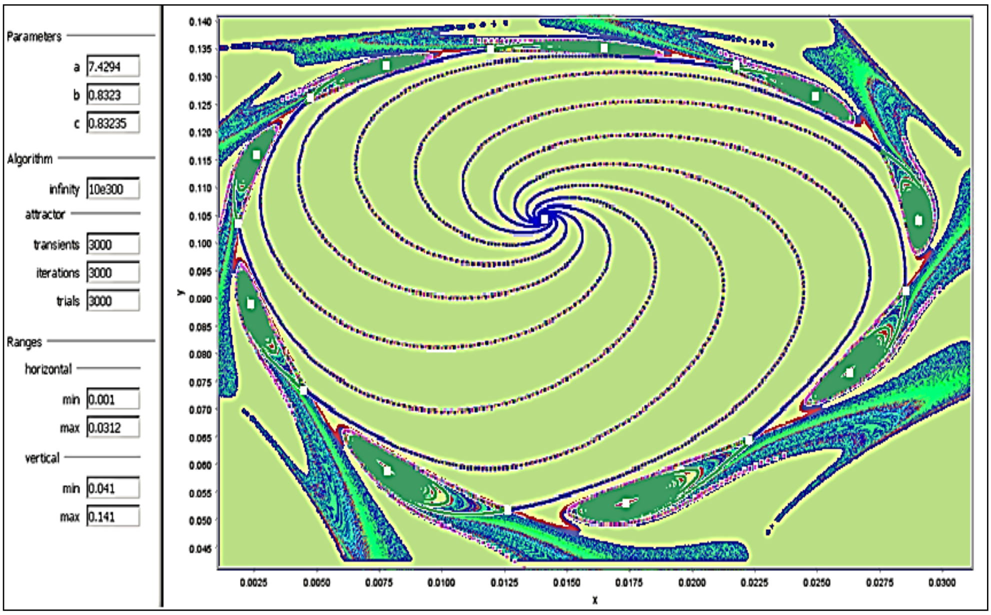

Figure 8. Nine saddles and nine focuses made in iDMC

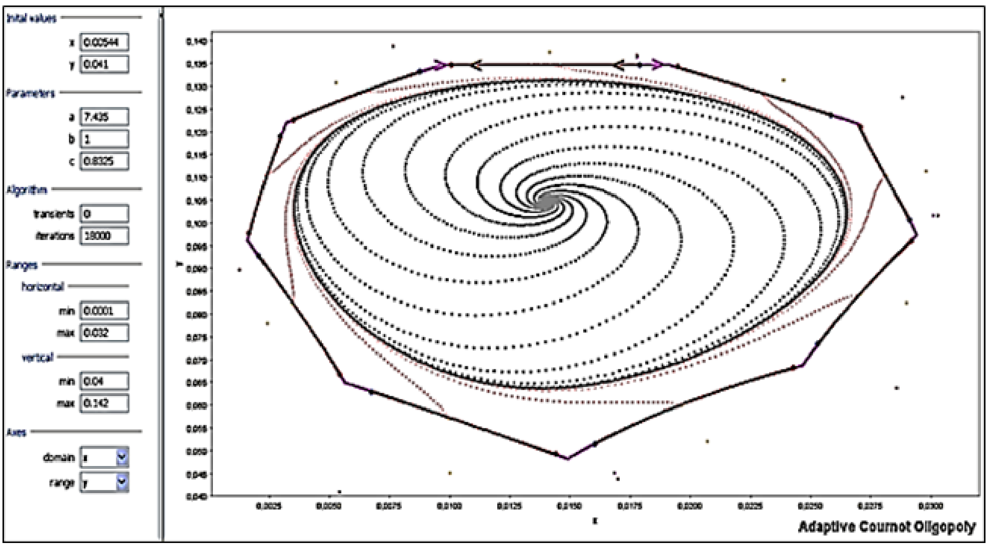

Figure 9. The repellent closed invariant curve and attractive orbit polygon

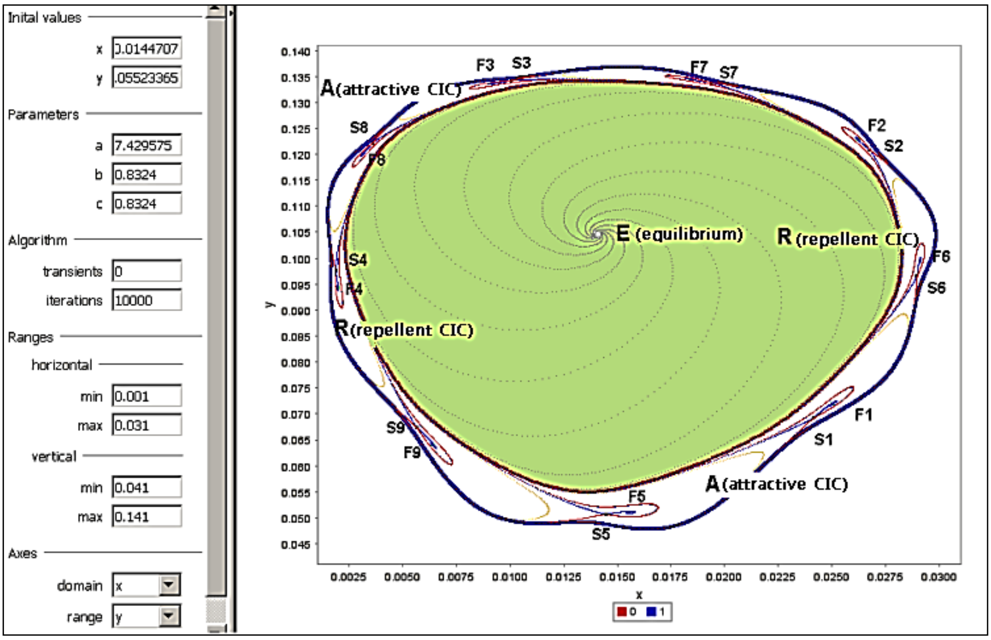

Figure 10. Further duopoly model with very complex behaviour

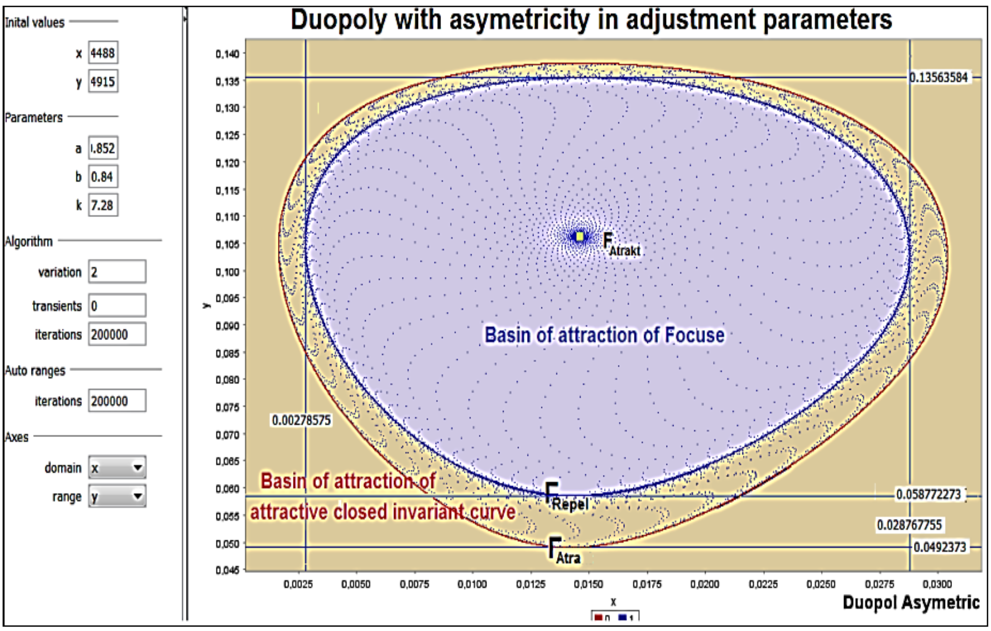

Figure 11. CIC´s: The impact of differences between adjustment parameters

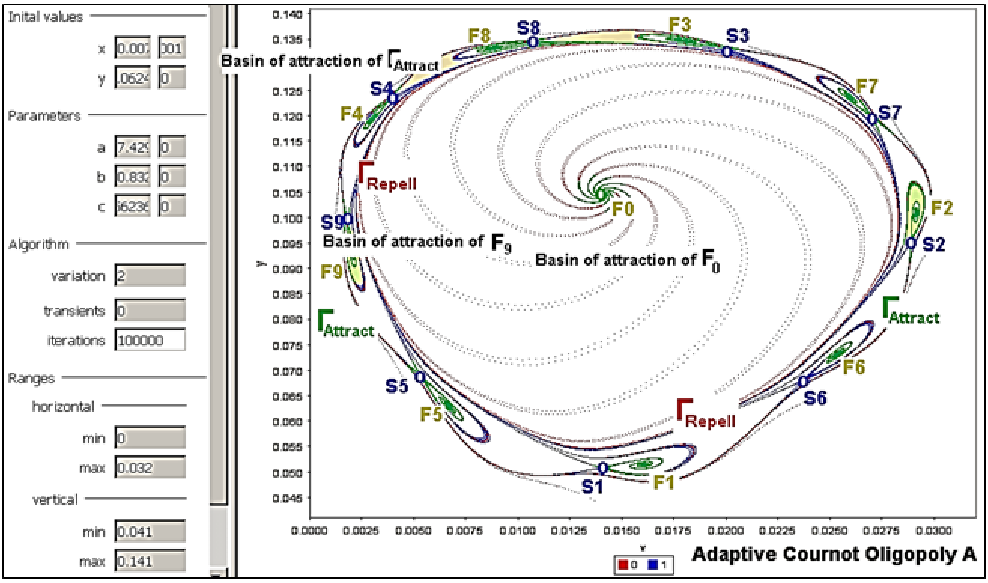

Figure 12. Other impact of different adjusting speed

Because the longitude of steps is differing, the state/phase point sometimes bounces/dances. The drunkards are faltering around. The Gemini's of nine polygon can to go for merging because shortening the distance between S_i and F_i (i = *1* … *9*) caused by changing of all of three parameters.

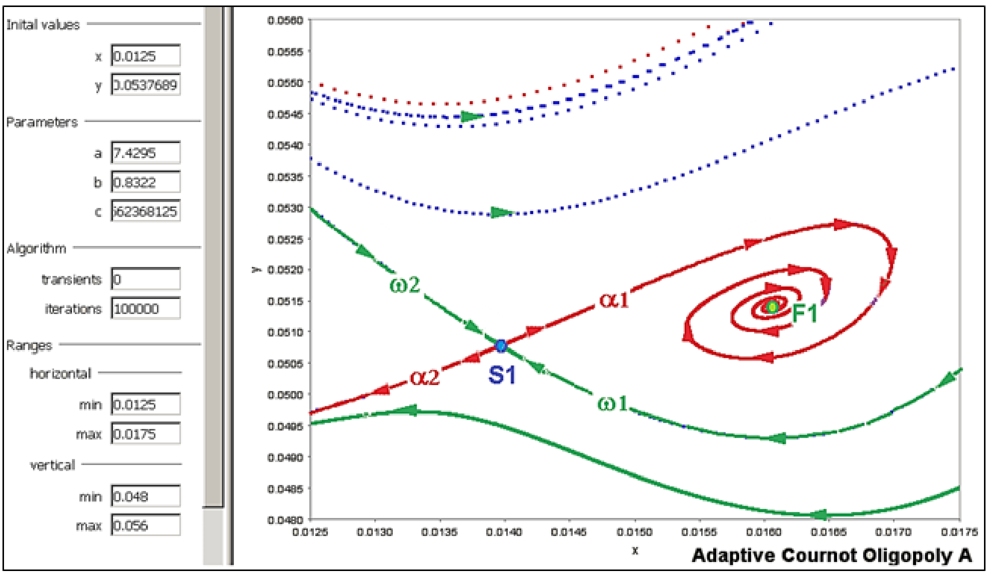

Figure 13. Chosen one of distant Gemini Saddle-Focus from polygon from nine ones

In the snapshot of Figure 13 we can see the two pair of saddle S1 branches (The details around Gemini S1-F1). The one of repellents is going to attractive focus *F1* that is $\alpha 1$, to second one $\alpha 2$ is attracted by CIC *GA*. The second pair of branches is attractive, that is *w1* and *w2*. They are coming from repellent CIC *GR*. The complexity of divorcing of attractive branches from CIC *GR* shows the snapshot of Figure 14.

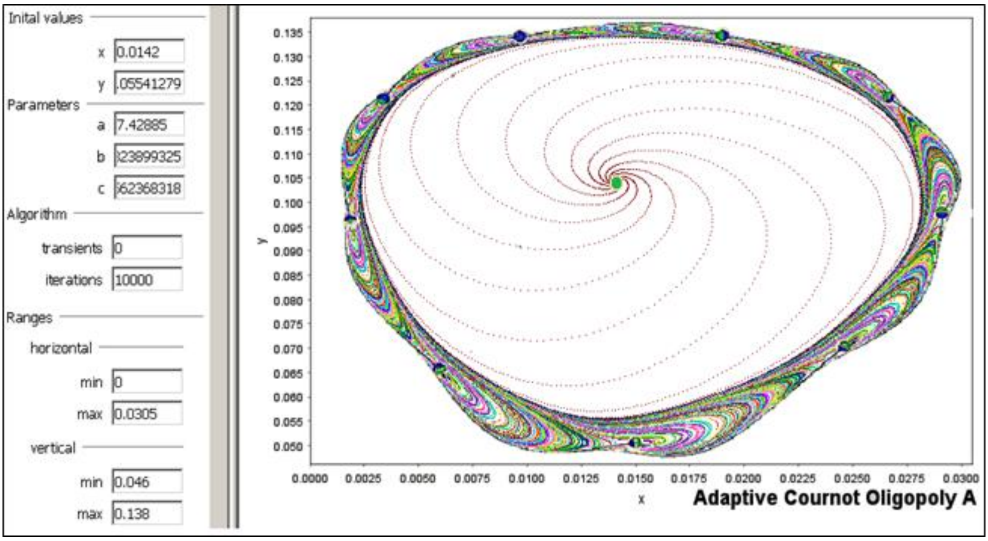

Figure 14. Complexity with two invariant curves and nine focuses

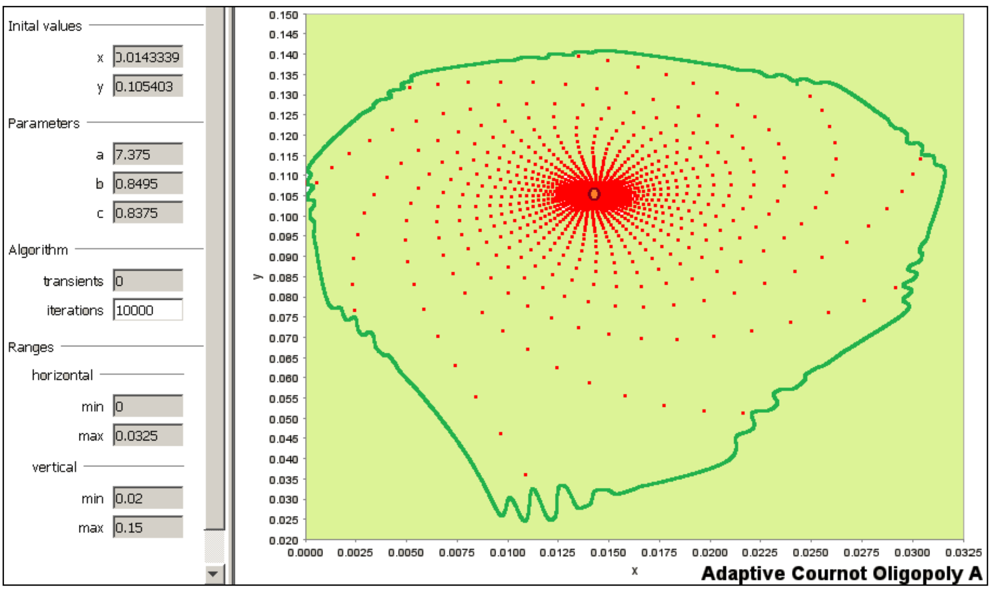

Figure 15. The repellent E* and attractive strange CIC

In a snapshot of Figure 14 there is the Cournot point as attractive focus in the centre. We can see also repellent closed invariant curve and attractive one. Between them there is a static polygon with nine edges, its nine points was united into saddle-node points.

It is need to note upon difference between cases of orbit and closed invariant curve. For example it is in the case of nine edge polygon of duopoly model. In the case of orbit the edge points are in stable loci whilst in the case of CIC whole polygon is rotating upon it’s one in opposite direction to clockwise movement. If the difference between adjustment parameters is increasing the behaviour of duopol is going to extreme. In snapshot of Figure 15 we can see strange attractive CIC *GA*.

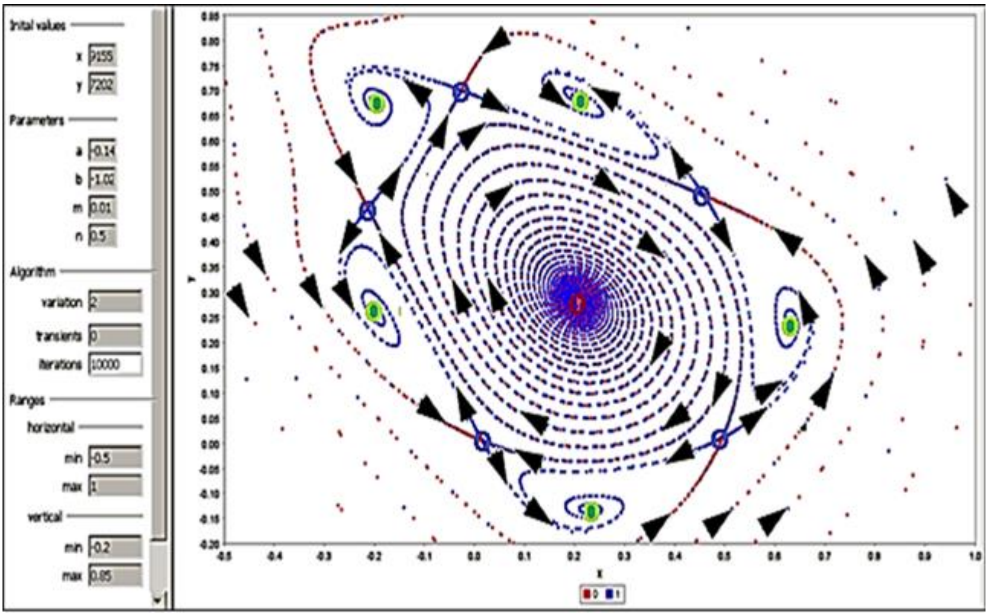

Figure 16. The phase point is winding around the five focuses:
everyone from two sides and two directions too

The central focus on Figure 16 is repellent with five branches, which are attractive branches of five saddles. Two repellent branches of each saddle are attracting either to left or to right neighbour focus in opposite directions.

2 The case of duopoly model with two mutually different products

In this section we are showing a duopoly complicated by differentiating of former identical product for two different ones. The consequences is that the market is becoming diversified to two areas for first and/or second goods (two-colour market). To get such duopoly model we are introducing parameters α and β, so that $\alpha + \beta = 1$, consequently $\beta = (1 - \alpha)$, (or in other sense parameter α gives the degree of substitutability/differentiation among the commodities). The new duopoly model (as topological map) becomes:

$$T = \begin{cases} q_1^{/} = q_1 + k_1\left(\dfrac{\alpha q_1^{\alpha-1} q_2^{\alpha} - c_1(q_1^{\alpha} + q_2^{\alpha})^2}{(q_1^{\alpha} + q_2^{\alpha})^2}\right) \\ q_2^{/} = q_2 + k_2\left(\dfrac{\alpha q_2^{\alpha-1} q_1^{\alpha} - c_2(q_1^{\alpha} + q_2^{\alpha})^2}{(q_1^{\alpha} + q_2^{\alpha})^2}\right) \end{cases}, \quad (4)$$

where “/” denotes the unit-time advancement operator, q_i is the quantity of different product, c_i is the production cost, and k_i is the “speed of adjustment”, respectively. Due to the presence of the denominator, it is obvious that T is defined only at points such that $(q_1, q_2) \neq (0, 0)$; furthermore from an economic point of view we are only interested in the study of the local stability properties of the positive output equilibria, i.e. points belonging to the positive quadrant of the plane R^2. In economic literature there are several similar models to map (4) but we were chosen it from outstanding essay of A. Agliari at all [1] because it’s very suitable character for our purposes. Map (4) is come into being by consecutive creative process. The utility function:

$$U(q1, q2) = q_1^{\alpha} + q_2^{\alpha}, \quad 0 < \alpha \le 1 \quad (5)$$

which is maximized subject to the budget constraint equation:

$$p_1 q_1 + p_2 q_2 = 1 \quad (6)$$

where p_1 and p_2 are the prices in the market of good q_1 and q_2 respectively and we assume the consumer’s exogenous income equal to *1*. Maximizing (5) subject to (6) results in the *demand functions*, that is in equation:

$$q_1 = \frac{p_2^{\gamma}}{p_1}\frac{1}{p_1^{\gamma} + p_2^{\gamma}}, \quad q_2 = \frac{p_1^{\gamma}}{p_2}\frac{1}{p_1^{\gamma} + p_2^{\gamma}}, \quad (7)$$

with $\gamma = \alpha/(1 - \alpha)$, or their inverses:

$$q_1 = \frac{p_1^{\alpha-1}}{p_1^{\alpha} + p_2^{\alpha}}, \quad q_2 = \frac{p_2^{\alpha-1}}{p_1^{\alpha} + p_2^{\alpha}}. \quad (8)$$

Clearly, if parameter $\alpha = 1$, then we give usual price equation for solitaire good (that is for isoelastic demand function the price is reciprocal to whole amount of goods supplied in the market, see first section)

$$p = \frac{1}{q_1 + q_2}. \quad (9)$$

On the base of map (4) we constructed virtual laboratory in iDMC with name *kópiaCourtNa* which is like this:

```
--@@
name = "kópiaCourtNa"
description = "See Model refs in user's guide"
type = "D"
parameters = {"k1", "alfa", "c1", "k2", "c2"}
variables = {"x", "y"}
function f(k1, alfa, c1, k2, c2, x, y)
   x1=x+k1*(((alfa*(x^(alfa-1))*y^alfa)-c1*((x^alfa)+y^alfa)^2)/((x^alfa)+y^alfa)^2)
   y1=y+k2*(((alfa*(y^(alfa-1))*x^alfa)-c2*((x^alfa)+y^alfa)^2)/((x^alfa)+y^alfa)^2)
return x1, y1
end
```

Using this laboratory we prepared simulation runs and several snapshots from them are shown in subsequence figures.

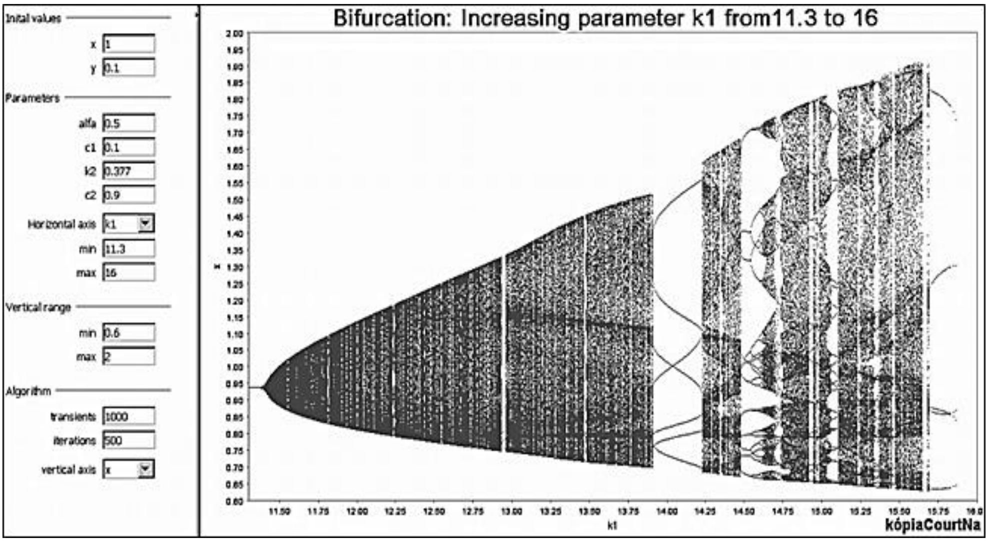

Figure 17. Single parameter bifurcations for k1

The great advance of iDMC is that we can do to run double bifurcation of parameter k_i. In Figure 19 we convey the situations with point I_1 and I_2. These points are intersections of two hyperbolas and one parabola which arcs is clearly seen in yellow colour. The most important and interesting area is area laying in right side of parabola arcs. There are a lot of so called Arnold's tongues, this area is named after its character of Neimark-Sacker bifurcations. Further snapshots which are following show details of double bifurcation made for k_i parameters.

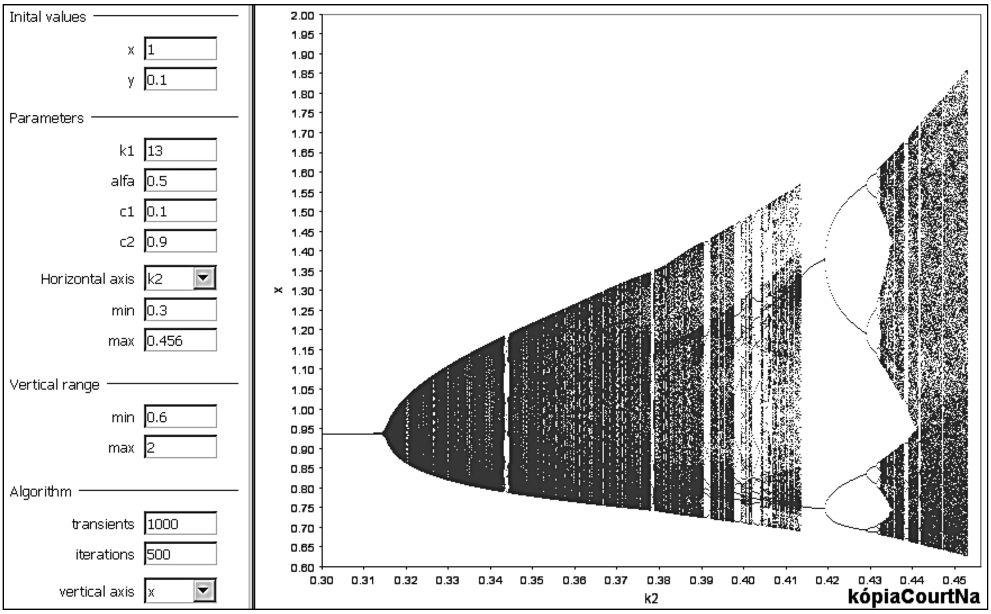

Figure 18. Single parameter bifurcations for k2

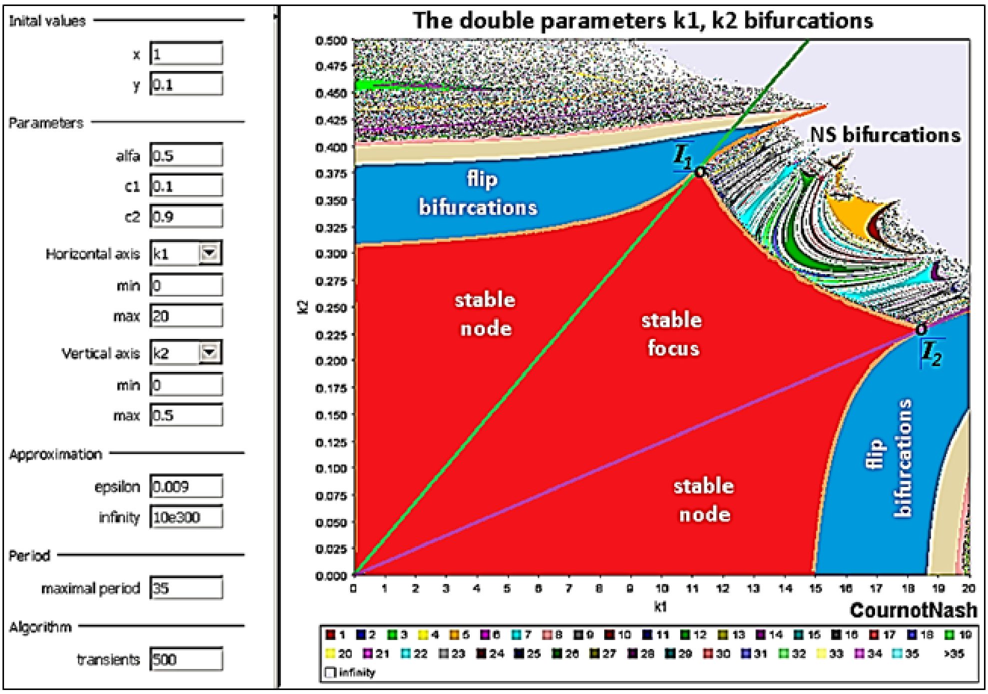

Figure 19. Situations in bifurcation area k1 & k2

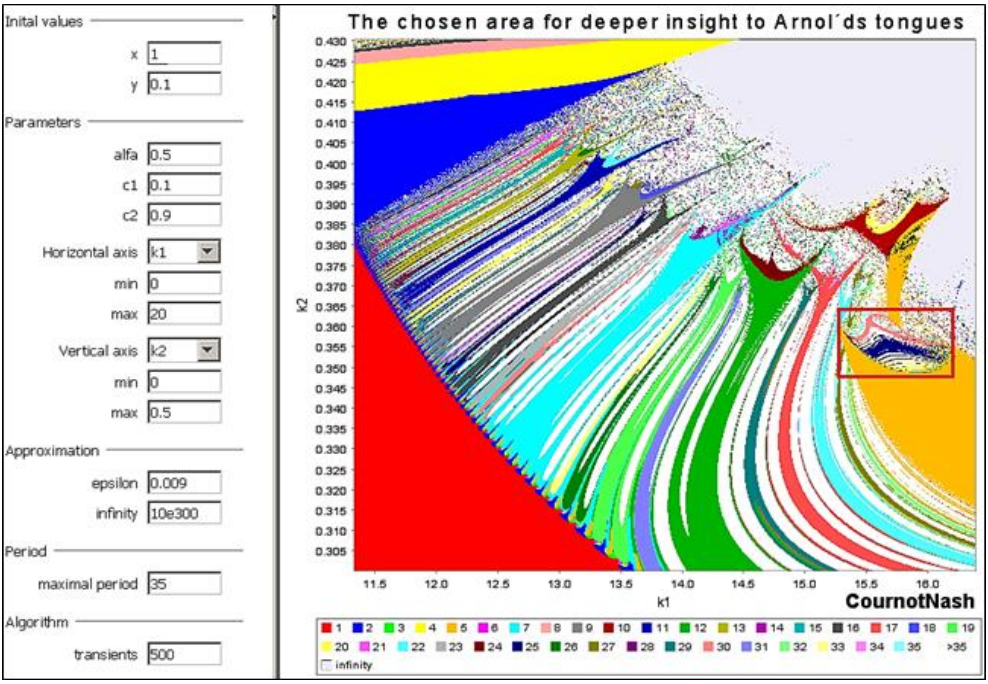

Figure 20. The Arnold´s tongues – detail

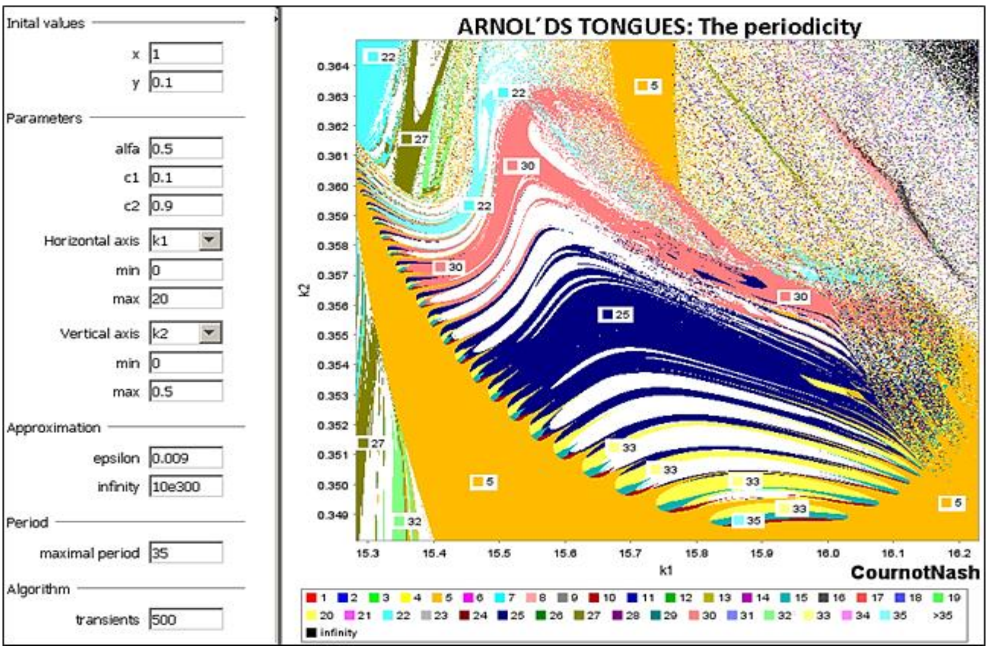

Figure 21. The deep insight into Arnold´s tongues (small window in Figure 19)

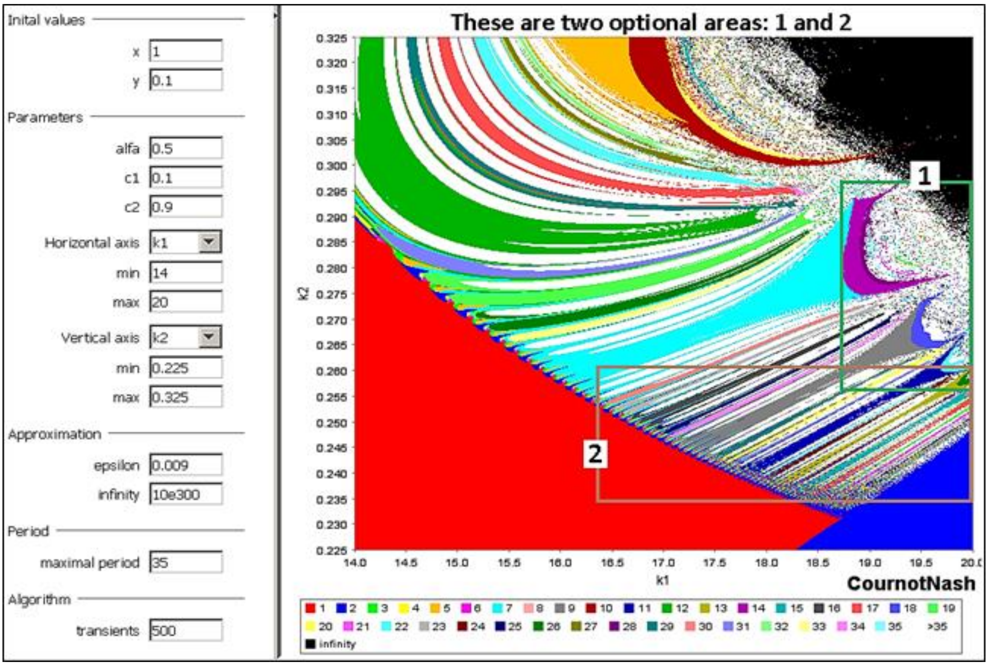

Figure 22. Two areas draft in the sphere of Arnold´s tongues (Neimark-Sacker) to choose

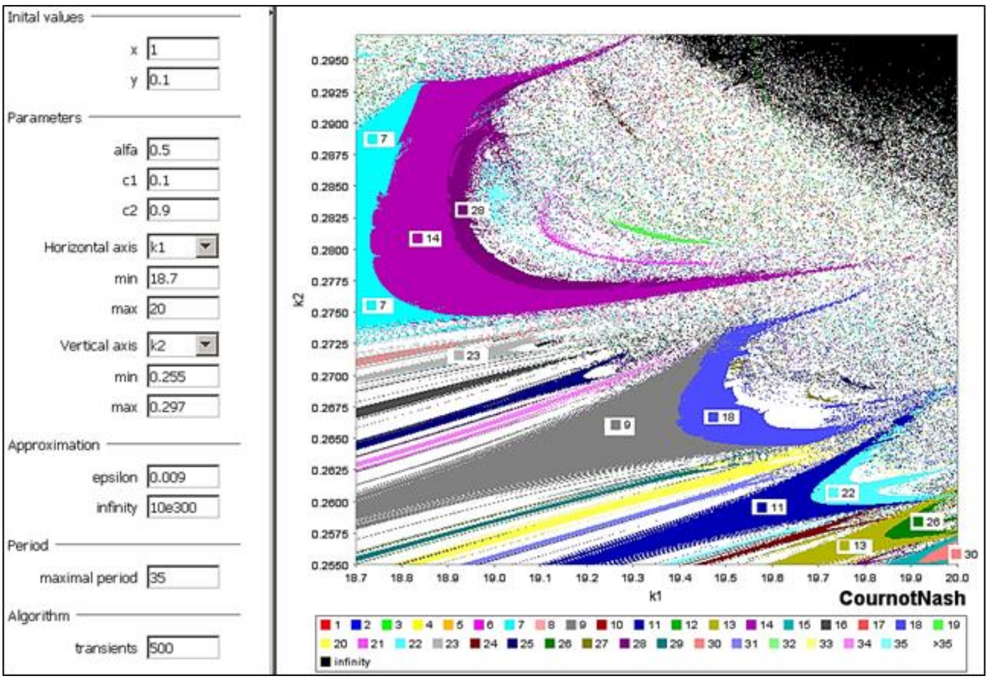

Figure 23. The cycles in chosen area No.1 as shown in Figure 21

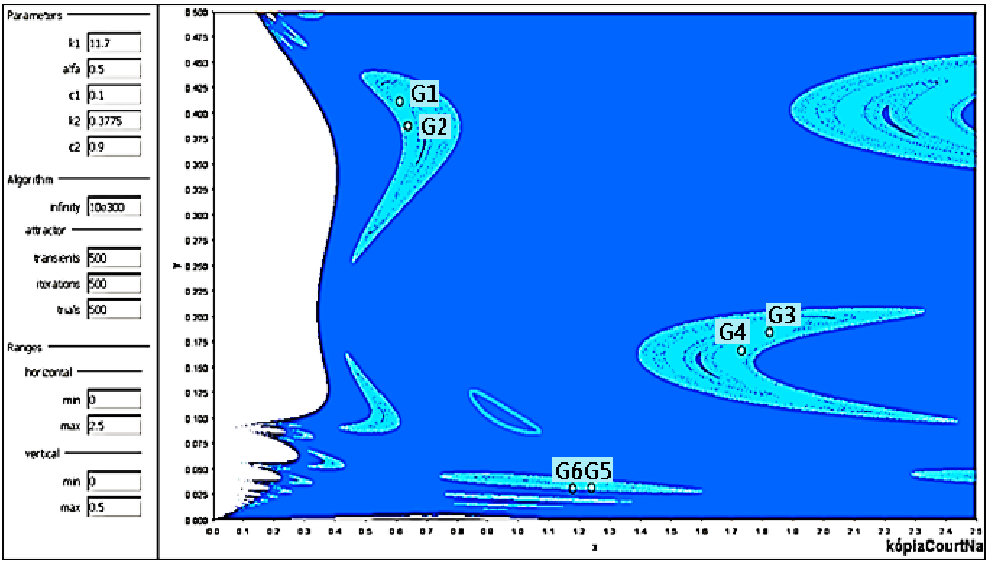

Figure 24. The closed invariant curve coexists with the stable 6-cycle G_i[3] (k_2 = 0.3775)

In the snapshot from simulations (different k_2´s) in iDMC laboratory the Figure 24 is visualising multistability in this type of duopoly dynamics. The Cournot point is in closed invariant curve and coexists with two attractors in those snapshots, unfortunately it is hidden.

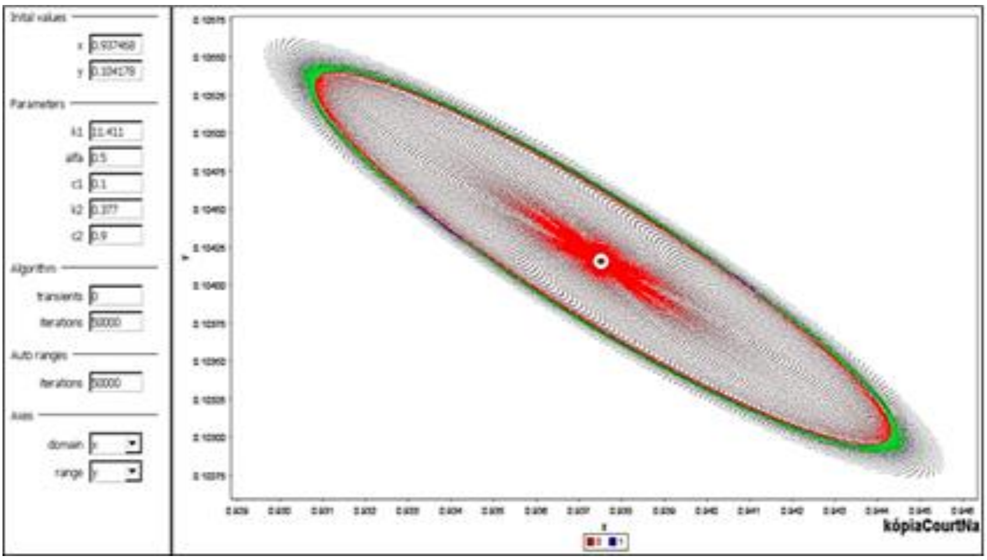

Figure 25. The attractive smooth CIC and repellent E*

[3] It is needed to note that three visualised basins of cycles G_i have a fractal structure.

Conclusion

The economy in objective reality is very complex evolutionary entity with strange dynamics. This object of study with such characteristics needs very sophistically elaborated ontology and the economic scholars maybe not meet this in the realm of conventional (and/or orthodox) economic approaches. The notoriously used ontology is dwelling very close to mechanics and classical thermodynamics. Economy is vertically layered multiplex network with complex behaving nodes and with tangled web of relations among them in all distances. Because sizes and behaviour of nodes and connections is in permanent change it can be envisaged as complex behaved creature. In nature such entities alike are coral shelves and tropical rain forests (jungle). Economy is one of the most complex system in the planet Earth. This is the reason why conventional economical approaches, methods and tools are bearing not adequate results. Our lot of years of experience with ICT/CI approaches to economic imagination and reasoning justified us to conclusion that every economist have to be aware not to create naïve imagination about behaviour of economy by conventional mode. It is most advantageous for them to use ICT/CI approaches.

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