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COMPARISON OF FIRST-PRINCIPLES AND EXPERIMENTAL VEHICLE MODELS

Dávid Mikle, Juraj Račkay

Abstract:

This article deals with the development and comparison of different vehicle models to be used in vehicle dynamics control of an electric all-wheel drive vehicle in the future. The aim is to verify the accuracy of lateral and longitudinal motions of selected vehicle models based on two different approaches. Using first principles describing basic vehicle dynamics, single and the twin track vehicle models are derived in form of systems of nonlinear differential equations. Using experimental identification, five vehicle state space models of orders are identified based on measurements on a real vehicle. The experiments performed were three different test maneuvers. Vehicle models are then compared using the measured data with the simulation results in MATLAB.

Keywords:

Vehicle dynamics, vehicle model, state-space model, MATLAB, systemIdentification toolbox.

ACM Computing Classification System:

Dynamic systems control.

1 Introduction

A significant challenge in electric vehicles with all-wheel drive is the way how to control whole powertrain to improve vehicle dynamics while increasing safety and stability. This can be achieved by various control strategies with different complexity. Each of these strategy is based on physical laws and mathematical descriptions of vehicle motion – vehicle model.

A wide range of simplified vehicle dynamics models is available in the literature, which are able to accurately represent basics of the force and moment dynamics [1]. However, in case of critical vehicle situations, it is essential to assume a more descriptive model considering the couplings between vehicle components. The application of multi-body dynamics for the analysis of vehicle handling problems was firstly discussed in [2], which was later applied in several complex multibody dynamics models in [3] and [4]. A complete multi-body model of the vehicle including the suspension geometry and tire characteristics was introduced in [5]. Simultaneously several multi DOF nonlinear multi-body dynamic models that can accurately represent almost all physical characteristics of the vehicle including suspension and tire dynamics has been proposed in [6] and [7]. But all these models have huge demands in computation power. To scale down the computational requirement, an intermediate multi-body model was proposed by authors of [8], which was well accepted in automotive industry for less critical applications.

However, these models are rarely used in their raw form, because of demand of simultaneously solve the combined differential or algebraic system of equations, which brings convergence issues with an even more computational requirements to the system.

Therefore a dominant amount of vehicle control strategies, for instance, the side slip control, yaw control, and trajectory control are based on a linearized version of vehicle operating condition known as the single track model. Example of step by step derivation of such a model can be found in [9]. In this model the vehicle is considered in its most simplified form, neglecting tire's side sliding, all lifting, rolling and pitching motion and assuming constant mass distribution on the axles. Of course, these simplifications come at the cost of reduced accuracy in the model as compared with actual vehicle motion, but model outputs still fits reality enough for purposes of vehicle control.

2 Vehicle Model Development

In vehicle planar movement analysis, with neglecting the internal forces, vehicle has 6 degrees of freedom (DOF). Number of DOF can be additionally reduced by several assumptions when considering only longitudinal, lateral and yaw motion. With used simplifications, in next chapters we consider vehicle of 3 DOF.

A Single track model derivation

Single-track model (Fig.1) describes well basic drive processes without much effort in modeling and parametrization. In this paper for single track model representation, we will take derivation of model from work of Efremov [9].

This model is used to describe planar vehicle motion, with next simplifications:

- All lifting, rolling, and pitching motion is neglected.
- Vehicle mass is concentrated at the center of gravity.
- Front and rear tires are represented as one single tire on each axle.
  Imaginary contact points of tires and surface are assumed to lie along the center of axles.
- Pneumatic trail and aligning torque resulting from a side-slip angle of a tire are neglected.
- Mass distribution on the axles is assumed to be constant.

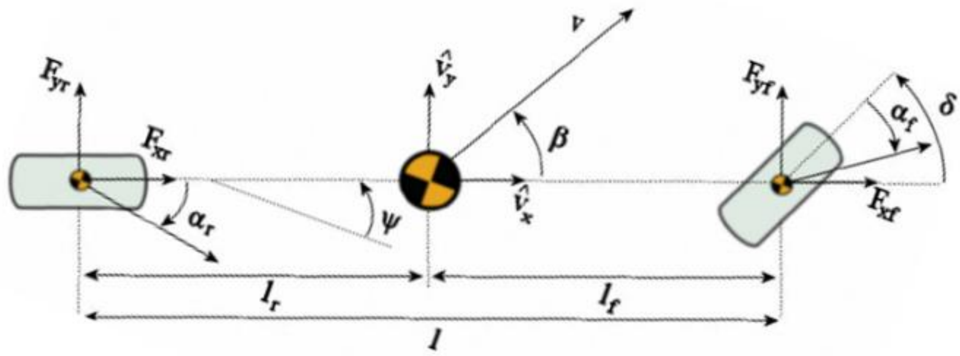

Fig.1. Single track model.

With assumed simplifications above, for considered vehicle three equations of motion exists:

$$F_x = -mv(\dot{\beta} + \dot{\Psi})\sin(\beta) + m\dot{v}\cos(\beta) \tag{1}$$

$$F_y = -mv(\dot{\beta} + \dot{\Psi})\cos(\beta) + m\dot{v}\sin(\beta) \tag{2}$$

$$M_Z = I_Z\ddot{\Psi} \tag{3}$$

Where, m is the vehicle's mass, v is the velocity of the center of gravity COG of the vehicle, β is the side-slip angle, ψ is the yaw angle, I_Z is the moment of inertia of the vehicle around the z axis. On the other side of the equations, there are forces acting on the COG of the vehicle along with x (F_X) and y (F_Y) axes and the moment acting around the z axis (M_Z).

The resulting system of nonlinear differential equations describing the steering angle projection and vehicle dynamics can be written as follows:

$$\dot{\beta} = -\dot{\psi} + \frac{1}{mv}\left(F_y\, cos(\beta) - F_x\, sin(\beta)\right) \tag{4}$$

$$\dot{v} = \frac{1}{m}\left(F_y\, sin(\beta) + F_x\, cos(\beta)\right) \tag{5}$$

$$\ddot{\psi} = \frac{1}{I_z}M_z \tag{6}$$

For this system sum of forces acting on vehicle in each direction can be derivated as:

$$F_x = F_{xf}\ \cos(\delta_f) - F_{yf}\ \sin(\delta_f) + F_{xr}\ \cos(\delta_r) - F_{yr}\ \sin(\delta_r) \tag{7}$$

$$F_y = F_{xf}\ \sin(\delta_P) + F_{yf}\ \cos(\delta_f) + F_{xr}\ \sin(\delta_r) + F_{yr}\ \cos(\delta_r) \tag{8}$$

$$M_z = l_f\left(F_{xf}\sin(\delta_f) + F_{yf}\cos(\delta_f)\right) - l_r\left(F_{xr}\sin(\delta_r) + F_{yr}\cos(\delta_r)\right) \tag{9}$$

Where δ_f , δ_r are steering angles of the front and the rear wheel, l_f , l_r is the distance from the vehicle's COG to the front and rear axle. Forces F_{Yf} , F_{Yr}, F_{Xf} , and F_{Xr} are forces acting on tires.

The tire dynamics is described by famous tire model Pacejka Magic formula, which can be used for estimation not only the lateral and longitudinal forces' impact on a tire, but also all the torques acting on a wheel around all axis. It has a straightforward calculation, and the same formula is used to estimate all the forces and torques using different sets of coefficients. The general Simplified Pacejka Magic formula has the following equation:

$$F = DF_Z\sin\left(C\ arctg(B\alpha - E(B\alpha - arctg(B\alpha)))\right) \tag{10}$$

Where D, C, B, and E is the set of shaping coefficients, F_Z is a wheel-load and α is tire side slip angle.

All these equation were implemented in single track Simulink model to verify performance of the derivated vehicle model.

B Twin track model derivation

For twin track model, in our previous work with Račkay [10], we developed Efremous single track model to cover all four wheels of vehicle. This model is based on the same three equations motion (1), (2) and (3), however in equation of acting forces (7), (8) and (9) we expand wheels elements from front *f* and rear *r* wheel to front right *fr,* front left *fl* and rear right *rr* and left *rl* wheels to cover remain dynamics. Expanded equations are in form:

$$F_x = F_{x_{PP}} \cos(\delta_{PP}) + F_{x_{LP}} \cos(\delta_{LP}) + F_{x_{PZ}} \cos(\delta_{PZ}) + F_{x_{LZ}} \cos(\delta_{LZ}) - F_{y_{PP}} \sin(\delta_{PP}) - F_{y_{LP}} \sin(\delta_{LP}) - F_{y_{PZ}} \sin(\delta_{PZ}) - F_{y_{LZ}} \sin(\delta_{LZ}) \quad (11)$$

$$F_y = F_{x_{PP}} sin(\delta_{PP}) + F_{x_{LP}} sin(\delta_{LP}) + F_{x_{PZ}} sin(\delta_{PZ}) + F_{x_{LZ}} sin(\delta_{LZ}) + F_{y_{PP}} cos(\delta_{PP}) + F_{y_{LP}} cos(\delta_{LP}) + F_{y_{PZ}} cos(\delta_{PZ}) + F_{y_{LZ}} cos(\delta_{LZ}) \quad (12)$$

$$\begin{aligned} M_z = l_P\{&F_{x_{PP}} sin(\delta_{PP}) + F_{y_{PP}} cos(\delta_{PP}) + F_{x_{LP}} sin(\delta_{LP}) \\ &+ F_{y_{LP}} cos(\delta_{LP})\} \\ &- l_Z\{F_{x_{PZ}} sin(\delta_{PZ}) + F_{y_{PZ}} cos(\delta_{PZ}) \\ &+ F_{x_{LZ}} sin(\delta_{LZ}) + F_{y_{LZ}} cos(\delta_{LZ})\} \\ &+ b_P\{F_{x_{PP}} cos(\delta_{PP}) - F_{y_{PP}} sin(\delta_{PP}) \\ &+ F_{x_{PZ}} cos(\delta_{PZ}) - F_{y_{PZ}} sin(\delta_{PZ})\} \\ &- b_L\{F_{x_{LP}} cos(\delta_{LP}) - F_{y_{LP}} sin(\delta_{LP}) \\ &+ F_{x_{LZ}} cos(\delta_{LZ}) - F_{y_{LZ}} sin(\delta_{LZ})\} \end{aligned} \quad (13)$$

Again all these equation were implemented in twin track Simulink model to verify performance of the derivated vehicle model.

3 Vehicle Model Identification

Vehicle model identification is an alternative process to model derivation, where you identify models with different representation from measured data. It is recommended to start by estimating the parameters of simpler models structures and if the model performance is poor, you gradually increase the complexity of the model structure. Ultimately, you choose the simplest model that best describes the dynamics of your system.

Vehicle itself is a complex systems with multiple inputs and multiple outputs (MIMO) and such systems are often more challenging to model because of couplings between several inputs and outputs. MIMO models are often covered via state-space representations, since the model structure complexity is easier to deal with [11]. State-space model use state variables to describe a system by a set of first-order differential or difference equations, rather than by one or more n^{th}-order differential or difference equations. The general state-space description has the following form:

$$\dot{x} = Ax + Bu \quad (14)$$

$$y = Cx + Du \quad (15)$$

(14) is called state equation with state vector *x*, (15) is output equation, *u* is input vector, *y* is output vector, coefficient *A* is system matrix, *B* is control input matrix, *C* is output matrix and *D* is feedforward matrix.

The state-space model structure is a good choice for quick estimation because it requires only one user input, the model order, n. In Matlab, model identification is supported in system identification toolbox [11] with user-friendly GUI.

In this work we identified five state space models of 3^{rd}, 4^{th}, 5^{th}, 6^{th} and 7^{th} model order from measured experiments.

4 Experiments

For experiments we used conventional vehicle with front wheel drive, equipped with multiple sensors. For position measurements was used self-build RTK GNSS receivers witch absolute accuracy of approx. 20mm, attached on each axle. We modified receiver's chips to be able to perform measurement at 20Hz frequency, what is in navigation systems pretty high performance. We also used a 9 DOF inertial measurement unit from XSENSE. As a vehicle output we measured accelerations and angular velocities in each directions witch frequency of 100 Hz. Position of used sensors is shown at following pictures:

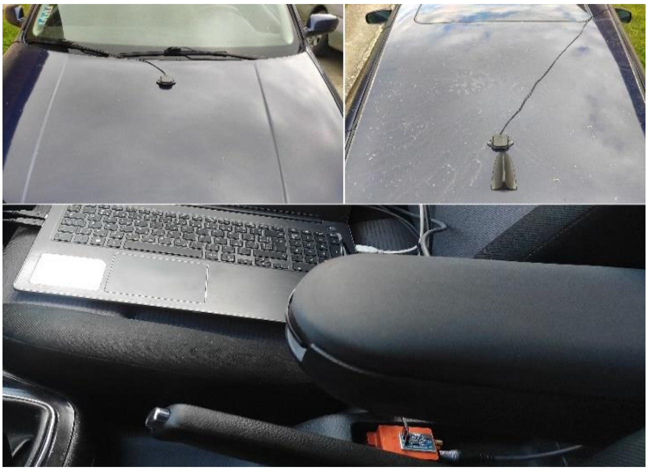

Fig.2. Position of GNSS antennas (Top), position of inertial measurement unit (Bottom).

In experiments we also performed a reading of CAN Bus of vehicle. It required a small intervention to vehicle's electric wiring, because a Volkswagen concern cars have switchable CAN bus on OBD2 connector, which means that it is not possible to read any data directly from it, because it requires a data polling from CAN Bus gateway. So we connected directly on CAN BUS wires in dashboard connectors and soldered wires to them. For data logging we used a self-build control unit with CAN transceiver with C++ library build for parse all data.

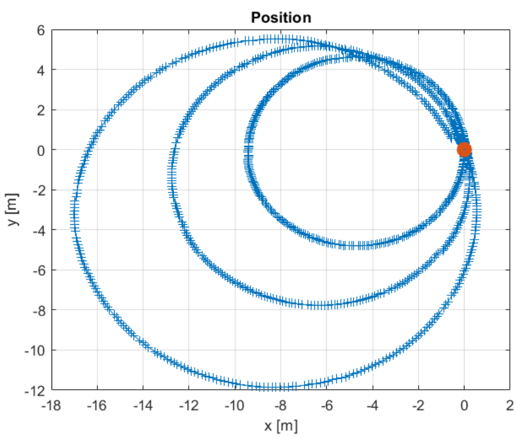

Fig.3. Trajectory of constant steer maneuver for multiple steering angles.

As experiments we performed a three different test maneuvers. First we performed a constant steer drive for different steering wheel angles at about 6 km/h, shown in (Fig.3). From trajectory of this maneuver we measured the real cornering radius for each steering wheel angle and from Ackerman geometry we calculated actual steering angles of the wheels. Then by polynomial regression we identified equation of steering wheel factor characteristic.

Next we performed a step steer maneuver (Fig. 4), where we accelerate with straight wheels from rest to about 30km/h and then applied a steering step to minus 15 degree. Data from this maneuver we used for model identification in MATLAB.

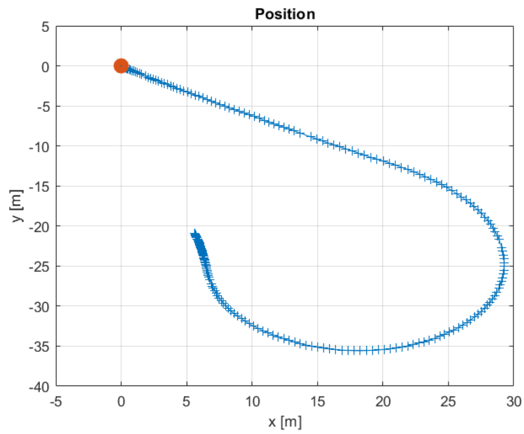

Fig.4. Trajectory of step steer maneuver.

Finally a third maneuver was single line change (Fig. 5), where we accelerate with straight wheels from rest to about 60km/h and the applied a minus 7degree input and immediately changed it to plus 7degree and then back to straight wheel. Data from this maneuver we used for verification of simulated models outputs.

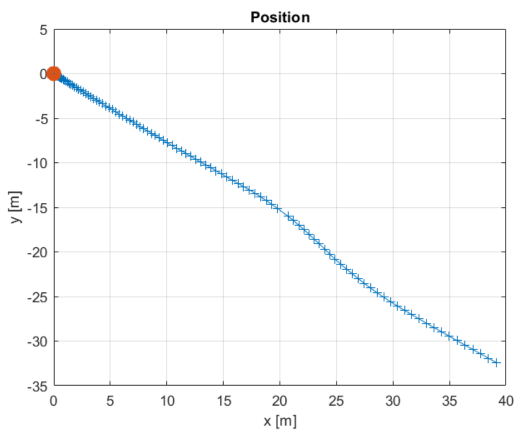

Fig.5. Trajectory of single line change maneuver.

5 Simulation

To verify derivated and identified vehicle models, we have simulated data from single line change experiment as test maneuver in MATLAB, to simulate the dynamic responses of models. The aim of this simulation is to verify the accuracy of lateral and longitudinal motion of models.

We simulated this maneuver with single track, twin track and identified state space models with 3rd – 7th order. Results of line change simulations are shown in the following figures.

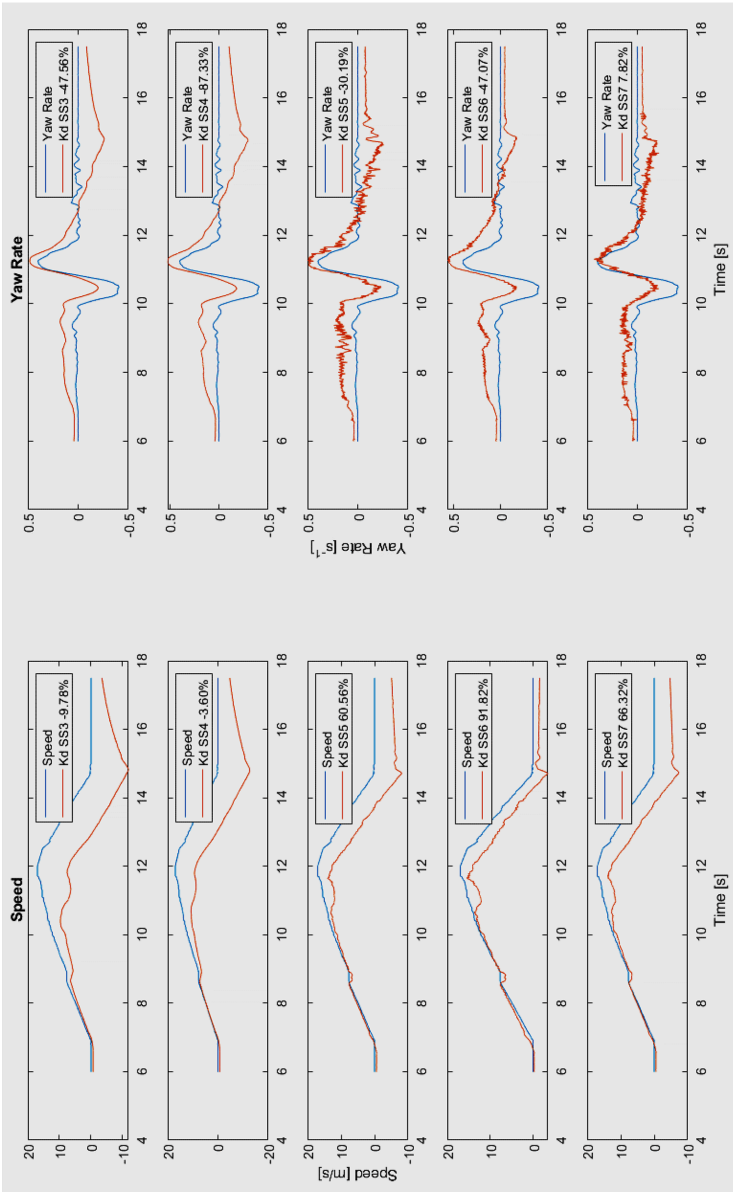

Fig.6. Comparison between measured vehicle speed (Left) and yaw rate (Right)
and simulated results from identified State Space model with 3rd order SS3(1st row), 4th order SS4(2nd row), 5th order SS5(3rd row), 6th order SS6(4th row) and 7th order SS7(5th row).

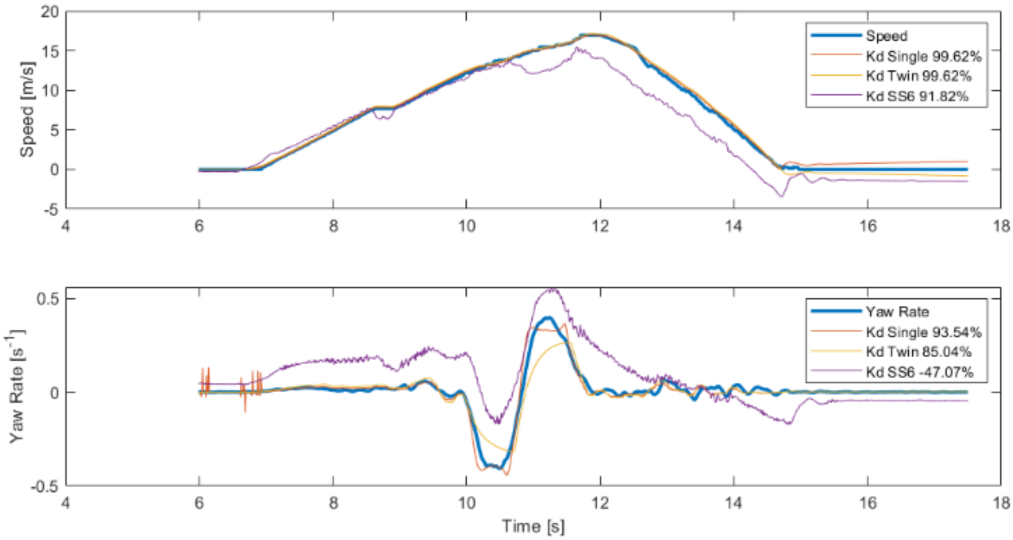

Fig.7. Comparison between measured vehicle speed (Top) and yaw rate (Bottom) and simulated results from single and twin track model and SS6.

From (Fig.6) with results of simulation of all identified models we can see that models with 3rd and 4th order (SS3, SS4) has a poor results, where coefficient of determination was negative number in both simulated outputs. Models with 5th, 6th and 7th order has significantly better results at simulated Speed, where model SS6 has Kd = 91.82%, what can be considered as great result. Also these models has slightly better result in simulated Yaw rate, however cause slight oscillations. In overall, from these results we can consider SS6 as a best identified state space vehicle model.

In (Fig.7) we can see comparison of model SS6, single and twin track model with actual measured data. From results we can see that derivated single and twin track model can even with many considered simplification pretty accurately represent longitudinal and lateral motion of vehicle. Coefficient of determination in simulated Speed for both models is approximately 99% and simulated Yaw rate is 85.05% for twin track and 93.54% for single track model.

6 Conclusion

The paper presents two most common vehicle model design approaches – a theoretical derivation based on first principles, and experimental identification, and their simulation-based comparison. In the design of any vehicle model, the most important objective is to obtain desired accuracy considering applied simplifications. Effectiveness accuracy of longitudinal and lateral motion of the developed models has been verified via line change simulations.

Based on simulation results, we can conclude that the developed single track and twin track vehicle models work as expected despite having used several simplifications in the theoretical background. The developed models are now ready to be implemented in the control system of the racing car of the Slovak formula student team STUBA Green Team.

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Authors

M.Sc. Dávid Mikle
Institute of Automotive Mechatronics
Faculty of Electrical Engineering and Information Technology
Slovak University of Technology in Bratislava, Slovakia
david.mikle@stuba.sk
He received the B.Sc. degree in Automotive Mechatronics and the M.Sc. degree in Applied Mechatronics and Electromobility from the Slovak University of Technology in Bratislava, Slovakia, in 2016 and 2018. During the M.Sc. degree, he was part of the slovak formula student team STUBA Green Team. Currently he is an interrupted Ph.D student in Mechatronic Systems and lecturer in subject Dynamics of Vehicles at the same university. His research interests include vehicle dynamics modeling and control, applied to electric vehicles.

MSEng. Juraj Račkay
Institute of Automotive Mechatronics
Faculty of Electrical Engineering and Information Technology
Slovak University of Technology in Bratislava, Slovakia
xrackay@stuba.sk
He received the Bc. degree in Automotive Mechatronics and the MSEng. degree in Applied Mechatronics and Electromobility from the Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Slovakia, in 2020 and 2022, respectively. Presently, he is a PhD student at the same university. His research interests include vehicle dynamics modelling and control of an electric vehicle traction system.