Application of the Hopfield Neural Network for Ring Balance Optimization

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ring resonator gyro, Hopfield neural network, static balance, dynamic balance, combinatorial optimization, permutation problem.

Abstract

For the first time, a problem of balance of the imperfect ring resonator gyro with inhomogeneous angular mass distribution density is considered as a discrete optimization problem and a neural network algorithm is proposed for finding its solution. The algorithm is based on minimization of the two-dimensional Hopfield network and is analogous to original algorithms for solving some problems of discrete optimization, such as the traveling salesman problem (TSP).

Introduction

The problem of static and dynamic balance of circular objects, such as a steam turbine wheel, an elastic ring resonator gyro, et al., arises from non-uniform angular mass or density distribution due to technological factors. Usually, parameters of this unbalance are known and, for example, it is possible to measure amplitudes and phases of harmonics of inhomogeneous distribution. Then, the balance is realized with the help of mechanical, laser, or other technologies. The simplest way to compensate the unbalance harmonics is to place or, alternatively, to remove unequal weights uniformly located at a priori determined places on the perimeter of the wheel (ring). The criteria of optimization is a total amount of the unbalance. So, the problem is to find an optimal permutation of the integers corresponding to a set of positive real numbers. This is a kind of combinatorial optimization problems for which solving no polynomial-time algorithms are known (the NP problems).

To increase the balance accuracy, large numbers of weights should be taken. So, the brute force or exhaustive search technique is not acceptable. Recently, a wide variety of meta-heuristic approaches, such as neural networks algorithms [1,2], simulated annealing method [3,4], genetic search algorithms [5,6], et al., were proposed for solving different classes of combinatorial optimization problems. In particular, in the work [7], a thermodynamically motivated (simulated annealing) optimization algorithm is used for balance optimization in connection with the problem of the static balance of a steam turbine circular wheel which has unequal weights of the turbine paddles.

In this paper, we consider an analogous problem for the sensitive element of the ring resonator gyro [8]. Here, for the first time we consider the latter problem as a problem of discrete optimization and propose the use of the Hopfield recurrent neural network for finding an optimal distribution of places where equally spaced weights on the perimeter of the elastic ring resonator must be removed. Unlike the paper [7], we solve not only the static balance problem, with aligning the center of masses, but also the dynamic balance problem, with simultaneous compensation of some first harmonics of a inhomogeneous mass distribution.

The paper is organized as follows. In the first part, we consider the principle of the ring resonator gyro operation. The second part contains the description of its balance and the mathematical model of the simplest point-wise balance procedure, which in practice can be realized with the use of the laser technologies. In the third part, the neural network algorithm of static and dynamic balance is described. Some comments and perspectives of our approach are given in the Conclusion.

Principles of Operation of the Ring Resonator Gyro

The ring resonator gyro belongs to the class of Coriolis vibratory gyros [8] whose operating principle is based on inertial properties of standing waves excited in elastic rings or shells. This effect was first discovered in 1890 by Bryan for bell-shaped resonators [9]. Due to the action of Coriolis forces, standing waves in objects rotating with angular velocity Ω precess both with respect to the resonator and in the inertial space.

For a long time the Bryan effect was not used in practice. Only in the middle of the 1960's the first angular velocity gyros were constructed. Usually, these devices were based on cylindrical or hemispherical sensitive elements [8]. Recently, ring resonators manufactured with the use of micromechanical technologies [10] began to be the basic elements of the new type of MEMS resonator gyros. Here we shall consider the basic principles of ring resonator gyros operation.

Equations for free oscillations of a perfect inextensible ring have the form (1)

$$\ddot{w}'' - \ddot{w} + \kappa^2 (w^{VI} + 2w^{IV} + w'') = 0, \tag{1}$$

where $w=w(\varphi,t)$ is the normal displacement of ring masses with respect to the angular coordinate and time; $\kappa^2 = EI/(\rho S R^4)$; ρ is the ring material density; S is the ring cross-section area; E is the Young's module; I is the moment of inertia of the cross-section with respect to the axis of bending; R is the radius of the unstrained ring.

In the system described by the Eq. (1), standing waves with own frequencies

$$\omega_k = \kappa \frac{k(k^2-1)}{\sqrt{k^2+1}}, \quad k = 2, 3, \ldots \tag{2}$$

can exist.

The case of the second mode excitation (Fig. 1) is most important in practice. Here the standing wave is represented by the following expression:

$$w(\varphi, t) = A(\cos 2\varphi_0 \cos 2\varphi + \sin 2\varphi_0 \sin 2\varphi) \cos \omega_2 t, \tag{3}$$

where A is the standing wave amplitude; φ_0 is the initial angle of the standing wave; $\omega_2 = 6\kappa/\sqrt{5}$ is the own frequency of oscillations.

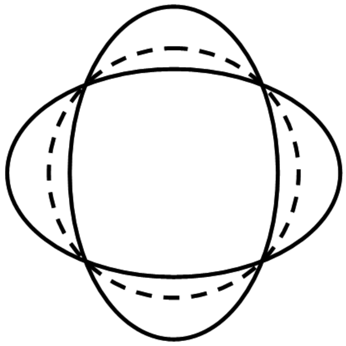

Figure 1. *The second mode of resonator oscillations*

If we begin to rotate the basement with angular velocity Ω (Fig. 2), the antinode orientation angle will rotate according to Eq. (4):

$$\varphi(t) = \varphi_0 - \frac{2}{5}\int_0^t \Omega(\tau)d\tau. \quad (4)$$

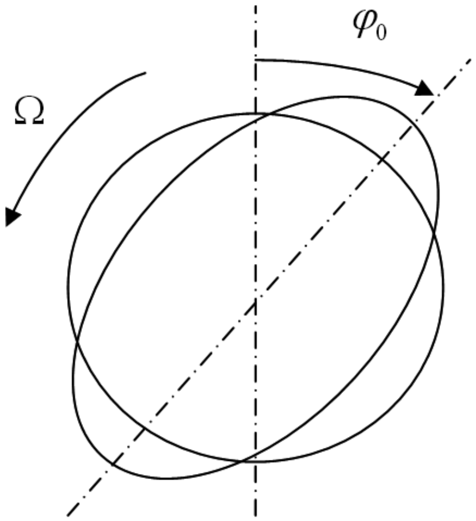

Figure 2. *Initial orientation of the standing wave excited in the ring fixed in rotating basement*

From Eq. (4) it follows that the angle of the standing wave rotation is less than the angle of basement rotation because, in the same conditions, the solid-state body rotates through an angle

$$\varphi(t) = \varphi_0 - \int_0^t \Omega(\tau)d\tau. \quad (5)$$

The factor

$$K = \frac{2}{5}$$

in Eq. (4) is called the precession coefficient of the ring resonator gyro.

Ring resonator balance

In practice, one should take into account different errors caused by mechanical, technological, temperature, and other factors. In imperfect systems one or more physical parameters may be inhomogeneous. Let us consider the case when material density is inhomogeneous with respect to the angular coordinate, i.e., $\rho = \rho(\varphi)$. Instead of Eq. (1) we have the following equation for own oscillations of the ring resonator:

$$\left[\frac{(\rho\ddot{w})'}{\rho}\right] - \ddot{w} + \kappa^2(w^{VI} + 2w^{IV} + w'') - \frac{\rho'}{\rho}\kappa^2(w^{V} + 2w''' + w') = 0. \tag{6}$$

Usually it is convenient to expand the density in Fourier series

$$\rho(\varphi) = \rho_0 + \sum_{k=1}^{K} \varepsilon_k \cos k(\varphi - \theta_\kappa), \tag{7}$$

where $\rho_0 = const$; ε_k and θ_k are amplitudes and angular orientations of harmonics.

In the general case, due to the oscillatory frequency splitting, it is not possible to excite standing waves in the unbalanced gyro. It was established that the frequency splitting caused by the fourth harmonic of the defect is proportional to the defect amplitude value, while the splitting for the first, the second, and the third harmonics is proportional to squared values of corresponding defects. Simplified scheme of mass defects is shown in Fig. 3.

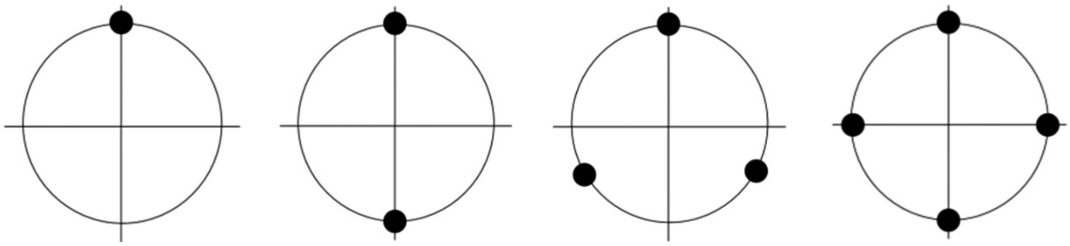

Figure 3. *The first four harmonics of inhomogeneous mass distribution*

Thus at balancing, we must pay the main attention to the fourth harmonics of the mass distribution defect. Here, the initially excited standing wave is destroyed and the oscillatory process is represented as the sum of two harmonic oscillations with different frequencies ω_{21}, ω_{22} (Fig. 4):

$$w(\varphi, t) = A(\cos 2\varphi_0 \cos 2\varphi\,\omega_{21} t + \sin 2\varphi_0 \sin 2\varphi\,\omega_{22} t). \tag{8}$$

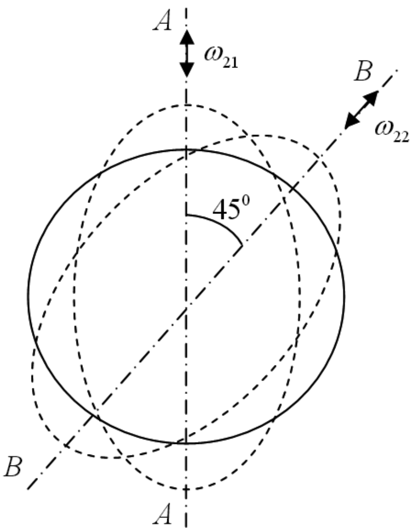

Figure 4. *Splitting of frequencies in the imperfect ring resonator*

Two main types of the balancing are distinguished: the static balancing providing coincidence of the mass center with the symmetry axis; the dynamic balancing for removing the frequency splitting. The balancing follows the measurement of effects caused by mass distribution anomalies. Then, these defects are compensated by means of the point-wise or distributed weights removing with using mechanical, laser, ion-plasma, or chemical technologies.

One of the main problems connected with the mechanical balancing is the localization of the segment of weights removing and the weights values to be eliminated. The number of such segments can be great enough because, for example, elimination of weight at one point on the resonator perimeter for removing the first harmonic of the defect can cause appearance of higher harmonics. To avoid this it is necessary to realize balance at many points with different values of eliminated weights.

Let us rewrite Eq. (7) in the form

$$\rho=\rho_0+\sum_{k=1}^{K}\varepsilon_k\cos k(\varphi-\theta_\kappa)=\rho_0+\sum_{k=1}^{K}d_{ck}\cos k\varphi+d_{sk}\sin k\varphi), \qquad (9)$$

where

$$d_{ck}=\varepsilon k\cos k\theta_k+d_{sk}\varepsilon_k\sin k\theta_k\,.$$

The general statement of the problem is as follows. Given *2K* real-valued positive numbers d_{ck}, d_{sk} *(k=1,…,K)*. We must find *2N* unknown parameters m_j, φ_j *(j=1,…,N)* from the following system of *2K* equations:

$$\begin{cases}\sum_{j=1}^{N} m_j\cos k\varphi_j=d_{ck}\\ \sum_{j=1}^{N} m_j\sin k\varphi_j=d_{sk}\end{cases}\quad (k=1,\dots,K) \qquad (10)$$

Using the criteria of the least mean-square error we get the minimization functional

$$\Phi=\sum_{k=1}^{K}\left[\left(\sum_{j=1}^{N}m_j\cos k\varphi_j=d_{ck}\right)^2+\left(\sum_{j=1}^{N}m_j\sin k\varphi_j=d_{sk}\right)^2\right]\to\min. \qquad (11)$$

If K=1 we shall solve the static balance problem or the center of masses stabilization. The dynamic balance problem occurs if K1.

The problem of finding the optimal set of parameters m_j, φ_j *(j=1,…,N) is multivariable with a number of possible local minima.*

The following variants of the point-wise balance can be considered.

1. Fixed angles: $\varphi_j=2\pi j/N\ \ (j=\overline{1,N})$. This model is appropriate for resonators with special balance "teethes" on their edges. The model, it its turn, has some variants.

   1.1 Initial ordered set of weights is given (the permutations problem):

   $m_j=m(j),\quad j=\overline{1,N},$

   $m_{\min}\le m_{j1}\le m_{j2}\le m_{\max}\quad npu\ \ 1\le j1<j2\le N$

   Here, the optimal permutation of indexes $p_j\,(j=\overline{1,N})$ and corresponding positions of weights to be removed $m(p_j)$ must be found.

1.2 Arbitrary weights:

$m_{\min} \le m_j \le m_{\max}, \quad j = \overline{1, N}).$

An optimal set of weights m_j must be found, which vary either with fixed discrete steps δ_m or continuously.

2. Arbitrary angles φ_j $(j = \overline{1, N})$.

2.1 equal weights:

$m_j = m = const, \quad j = \overline{1, N}.$

Here, one should find arbitrary angles φ_j which vary either with fixed discrete steps δ_φ or continuously.

2.2 Arbitrary weights:

$m_{\min} \le m_j \le m_{\max}, \quad j = \overline{1, N}.$

In this model of the balance we must find optimal sets of weights m_j and angles φ_j, which vary either with fixed discrete steps δ_m, δ_φ, respectively, or continuously. This is the most complicated optimization problem in comparison with the above mentioned problems. In [11,12] some algorithms for solving the balance problem 2.2 are presented for a small number of compensated harmonics (K=1,2,3,4).

We shall consider a particular case 1.1 when the weights to be removed have fixed values m_j, $j = 0,1,\ldots, N-1$ and must be located at fixed angular positions, i.e., in the nodes of the regular mesh (Fig. 5)

$$\varphi_j = jh, \; j = 0,1,\ldots,N-1, \; h\frac{2\pi}{N}.$$

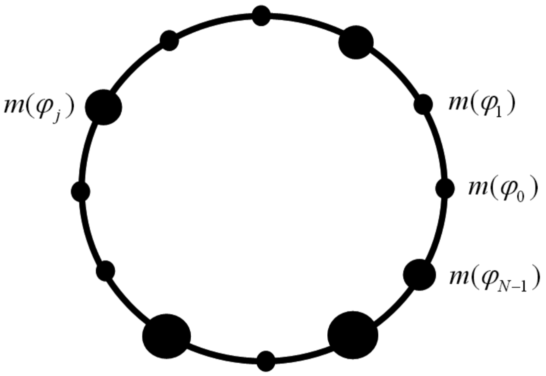

Figure 5. *Angular distribution of weights to be removed*

It is not difficult to note that the total number of possible solutions, up to a symmetrical permutations, is equal to *N!(2N)*.

Approaches considered earlier [11,12] did not propose in the general case the single algorithm for compensation of harmonic components of angular mass distribution. Some methods of artificial intelligence, such as neural network algorithms, genetic algorithms [13], simulated annealing technique [7], etc., could be useful for developing the universal balance algorithm.

Balance of the ring resonator by means of the Hopfield recurrent neural network

Let us consider a possible the neural network algorithm for balance of the imperfect resonator with inhomogeneous angular mass distribution density. It is based on minimization of the two-dimensional Hopfield network (Fig. 6) and is analogous to original algorithms for solving some problems of discrete optimization, such as the traveling salesman problem (TSP) [1,2] etc.

For formalized description of the problem, we introduce the Boolean variable y_{ij} which is equal to unity if the weight number i is removed in the angle number j, and equal to zero otherwise.

According to the statement of the problem the following restrictions must be observed:

$$\sum_{i=1}^{n} y_{ij}=1 \quad \forall j \in \overline{1,n}, \quad \sum_{i=1}^{n} y_{ij}=1 \quad \forall i \in \overline{1,n}, \tag{12}$$

where

$$y_{ij} \in \{0,1\},\ i,j \in \overline{1,n}.$$

Since the energy of the Hopfield network tends to minimum and, according to the statement of the problem, we must minimize the goal function which is the linear combination of the syntax function of the problem, minimal (equal to zero) if and only if each weight is located at one point and at each point only one weight is located, and the quality function (mean squared error with respect to all harmonics), then the correspondence can be established between them.

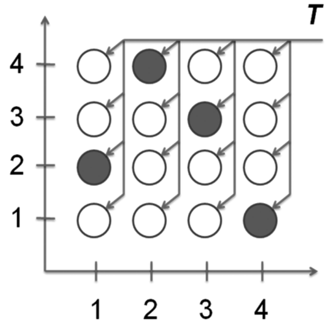

Figure 6. *Two-dimensional Hopfield recurrent network with active neurons (dark color, y_{ij}=1) corresponding to a possible "rook placement"; T is the threshold signal*

According to the mentioned above correspondence let us interpret restrictions and the goal function. As a result we get

$$F(y)=F_1(y)+F_2(y), \tag{13}$$

where $F_1(y)$ is the syntax function; $F_2(y)$ is the solution quality function.

The syntax function reflects the fact that in each angle strictly one weight must be removed. It may have various forms, for example, forms, analogous to that used by Hopfield for solving the TSP problem:

$$F_1(y)=\frac{a}{2}\sum_{\mu}\left(\sum_{i} y_{\mu i}-1\right)^2+\frac{b}{2}\sum_{i}\left(\sum_{\mu} y_{\mu i}-1\right)^2 \to 0. \tag{14a}$$

or

$$F_1(y)=\frac{a}{2}\sum_{\mu}\sum_{i}\sum_{j\neq i}y_{\mu i}y_{\mu j}+\frac{b}{2}\sum_{i}\sum_{\mu}\sum_{v\neq\mu}y_{\mu i}y_{vi}+\frac{c}{2}\left(\sum_{\mu}\sum_{i}y_{\mu i}-n\right)^2\to 0, \qquad (14b)$$

This function achieves minimum (zero) value if each angle corresponds to one weight to be removed and each weight to be removed is located in one angle.

The goal function characterizes the quality of a solution and has the following form:

$$F_2(y)=g\sum_{k=1}^{K}\left[\sum_{j=1}^{N}m(j)\exp\left(i\frac{2\pi}{N}jk\right)-\varepsilon_k\exp(ik\theta_k)\right], \qquad (15)$$

where $\theta_k=arctg(d_{sk}/d_{ck})$ are angular orientations of the defect harmonics.

In Eqs. (14), (15) a, b, c, and g are positive real-valued weight factors (usually it is recommended to take a=b).

The recurrent Hopfield network parameters are obtained if we establish correspondence between function (10) and the energy function of the Hopfield network. The latter is written as [1,2]:

$$E(y)=\frac{1}{2}\sum_{\mu}\sum_{v\neq\mu}\sum_{i}\sum_{j\neq i}w_{\mu ivj}y_{\mu j}y_{vj}+\sum_{\mu}\sum_{i}T_{\mu i}y_{\mu i} \qquad (16)$$

Equating coefficients at linear and quadratic terms in expressions for $E(y)$ and $F(y)$, we obtain weights of the Hopfield network. In the case of Eq. (14a), matching the linear terms gives us thresholds and matching the quadratic terms give us synaptic weights:

$$\begin{aligned}w_{\mu ivj}&=-a(\delta_{ij}+\delta_{\mu v})-bm_\mu m_v\sum_{k=1}^{K}\cos\frac{2\pi}{N}k(i-j),\\ T_{\mu i}&=-2a-2m_\mu\sum_{k=1}^{K}\varepsilon_k\cos\left(\frac{2\pi i}{N}-\theta_k\right).\end{aligned} \qquad (17)$$

The Hopfield recurrent neural network dynamics equation has the form

$$y_{\mu i}(t+1)=f\left(\sum_{v}\sum_{j}w_{\mu ivj}y_{vj}(t)-T_{\mu i}\right), \qquad (18)$$

where the step function

$$f(x)=\begin{cases}1, & x>0,\\ 0, & x<0.\end{cases}$$

Numerical example. Numerical experiments were performed to prove efficiency of our technique. Let us take M=N=12. The set of weights is as follows: 1,2,3,4,5,6,7,8,9,10,11,12, and, for simplicity, in Eq. (9) all $\varepsilon_k=0$. The Hopfield network found the following permutation of weights: 6,7,3,10,2,11,5,8,4,9,1,12. Here, due to Eq. (11), the static unbalance with respect to the first harmonics (the center of masses unbalance) was about $6.4\cdot10^{-15}$. Remained unbalance with respect to the second, third, and fourth harmonics were equal to 6.9, 4.2, 12, respectively.

If we want to pay the main attention to compensation of the particular harmonics, we just use weight coefficients in Eqs. (15), (17) before corresponding terms. Thus, in our example, if we want to compensate the first and the fourth harmonics, the Hopfield network gives us the following result permutation: 7,9,10,4,2,8,5,12,3,11,1,6. Unbalance values for the first four harmonics are 0.9, 16.1, 7.2, and 3.0, respectively.

It should be noted that the neural network algorithm demonstrate fast convergence. Outputs were stabilized after not greater than 5-10 iterations.

If we take a sufficiently large number weights, the network output often gives us solutions which do note obey the problem syntax (the "rook placement"). In connection with this, the following alternative statements of the balance problem as a combinatorial permutations problem can be considered.

1. It is admissible to place some weights in one angle. Here, we omit the restriction of "rook placement" when each weight to be removed corresponds strictly to one angle.
2. The first variant can be extended to the case when the number of weights *M* is greater than the number of angles *N*.
3. The first variant can be extended to the case when the number of weights *M* is sufficiently greater than the number of angles *N* and these weights are equal.
4. The number of weights *M* is less than the number of angles *N*. This assumption allows modeling inhomogeneous weights distribution when some angles with absent weights correspond to zero weights to be removed.

To achieve better results of the balance, sometimes it is advisable to adjust weights by means of local optimization with the use of one of gradient techniques.

Conclusion

In the article, for the first time we consider the balance procedure of the ring resonator gyro as a discrete optimization problem. Unlike some earlier proposed iteration and analytical balance techniques, the new approach, due to its flexibility, gives the possibility to realize different statements of the problem. A variant of combining the Hopfield neural network with simulated annealing technique can considered (the Boltzmann machine), which partially allows avoiding the local minima problem. Main results of the paper can be extended to the balance procedure of other Coriolis vibratory gyros (hemisphere resonator gyro, cylindrical resonator gyro, et al.).

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