Algorithmization of interaction of components of expert virtual resource of procedural type in managerial decision-making optimization

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Abstract:

For intellectualization of managerial decision making support in corporate systems it is recommended to use expert and virtual resource (EVR), which is a combination of expert and computer resources. In the case when knowledge corporate environment is poorly structured and it has obscure connections and a multi-level subordination hierarchy, the EVR of procedural type intended for intellectual support of decision-making, is used.

Considering the existing diversity of EVR of procedural type, there is a necessity to develop algorithms for interaction of their components in managerial decision making in various modes.

Algorithmization of a virtual mode of interaction is related to the sequential use of all the three types of virtual experts, thus the basic procedure of multialternative optimization used by multialternative virtual expert (MVE) prevails.

The principal task of developing the algorithm of dual mode interaction between the virtual and real experts in the environment of managerial decision making consists in development of a matrix game.

Algorithmization of a collective mode of interaction of real and virtual experts is based on a man-machine procedure which provides a dialogue with real experts on the basis of automatically offered questions, with further formalization of expert answers. An iterative principle of building of this procedure by means of immersion of question-answer process with a team of real experts in randomized environment, is proposed. A Team mode with a dominating expert is based on agreeing the assessments of priority of alternative optional managerial decisions by that expert with assessments by a team of peer experts. The developed algorithms can be integrated into the single environment of the managerial decision making subsystem and used as a means of intellectual support.

Keywords:

Optimization, simulation, managerial decision making, choice of dominant options, expert and virtual resource.

ACM Computing Classification System:

Parallel programming languages, Optimization algorithms, Exact arithmetic algorithms, Hybrid symbolic-numeric methods.

Introduction

The optimal managerial decision making is based on systemization, analysis of large amounts of accumulated information, use of methods of forecasting and mathematical modeling, engagement of leading experts in evaluation. As can be seen from the above, this process requires attraction of extensive resources both human (expert), and computer, and their close interaction. The set of expert and computer resources in a series of works is called expert and virtual resource (EVR) of decision making. [1-4]. It is proposed to identify the two types of EVR which are procedural and knowledgable. The first virtual expert resource provides intellectual support for decision choosing, the second supports all three stages of decision making.

Virtual expert resource of procedural type is meant to intellectualize support for managerial decision making in cases when the corporate knowledge environment is weakly structured, features fuzzy links and a multi-level subordination hierarchy.

The basic components of virtual expert resource are real and virtual experts, the basic principles of their interaction proposed in studies [5, 6].

Either an individual real expert (IRE) or a team of real experts (TRE) is brought in to make decision.

In their turn, virtual experts may be divided into the following types according to the functions they execute during decision making:

– imitational prognostic virtual expert (IPVE);

– multi-alternative virtual expert (MVE);

– multi-agent virtual expert (MAVE) [1-4, 7-9].

To formalize the interaction of components of the expert and virtual resource of a procedural type in making optimal managerial decisions, the problem of development of appropriate algorithms arises.

1. Virtual interaction mode

Algorithmizing the virtual mode of interaction involves sequential use of three types of virtual experts: IPVE, MVE and MAVE. The basic procedure is multi-alternative optimization used by MVE [5, 6]. It calculates variations of the criteria *F* and functions and constraints φ while the alternative variables change

$$z_m = \begin{cases} 1, \\ 0 \end{cases}, m = \overline{1..M} \ .$$

The variation of a given factor F over variable z_m at *k*-th iteration is given by:

$$\Delta_m^k F = F(\tilde{z}^k / z_m = 0) - F(\tilde{z}^k, z_m = 1),$$

where $z^k = (z_1, .., z_\nu, .., z_M)(\nu = \overline{1, M}, \nu \neq m)$ is the vector of random realizations of alternative variables. To organize the process of multi-alternative optimization and formation of sets of dominating options W_∂, three variations are used, i.e. 6 calculations of the factor are made.

Each calculation includes searching for the value of parameter f_m corresponding to the alternative variable $z_m = 1$. Next one has to retrieve the dependence of factor F on f_m: F= F(f_m). Calculations have to be carried out for indicators I_1 corresponding to the criteria, and I_2 corresponding to constraints. Therefore, the total number of search and retrieval procedures for each *k*-th iteration is

$$\pi^k = 6[\pi_n(z_m = 1, I_1 + I_2) + \pi_в(z_m = 1, I_1 + I_2)],$$

where the first element in the sum gives the number of search procedures of the first type depending on the number of alternative variables and the total number of criteria and constraints, and the second gives the number of retrieval procedures that depends also on the number of alternative variables and the total number of factors.

As for retrieval procedures, IPVE is used, and addressing the corporate intellectual capital is again needed to search for retrospective quantitative information and update expert data, as described in modeling techniques presented in Section 3.1. Note that one needs $\pi_n^в$ search procedures of second type to retrieve F=F(f_m) – $\pi_n^в$.

To implement search procedures of first type one uses the multi-agent virtual expert of first type with *x* agents, and that of second type with *y* agents. To forecast the degree of competition between the two types of MAVEs for the virtual expert resource, we use the system of iteration equations from Section 3.3:

$$\begin{cases} y^{k+1} = \alpha x^k (1 - x^k) \\ x^{k+1} = \beta y^k (1 - y^{k+1}) \end{cases}.$$

To assess the controlling parameters α and β we refer to the first two iterations finding $\pi_n^{(1)}$, $\pi_n^{(2)}$, $\pi_n^{в(1)}$, $\pi_n^{в(2)}$ for them and then retrieving x^1, x^2, y^1, y^2.

$$\alpha = \frac{y^2}{x^1(1 - x^1)},$$

$$\beta = \frac{x^2}{y^1(1 - y^2)}.$$

With α and β known we model numerically the system of iteration equations for k>2. In case we reach area 2, the number of agents in MAVEs of two types stabilizes, and it is feasible to proceed to the virtual interaction mode. Otherwise one needs to substitute one of the MAVEs. Selecting the two types of MAVEs goes on until we reach area 2 in the result of numerical modeling.

This sequence of operations constitutes selection of the more effective agent for search and assembly of information (SASA). Upon addressing SASA and defining the two types of MAVE, one needs to assess the number of iterations (k^0) needed to form the set of dominating options W_∂.

Functions belonging to the conditions of information balance for multi-alternative

optimization procedures [5] are found from experimental studies:
the current random amount of information in averaged message

$$\tilde{\varphi}_1(k) = \tilde{a}_{01} - \tilde{a}_{11}(1 - l^{\tilde{a}_{21}k});$$

and the current random average of channel throughput capacity

$$\tilde{\varphi}_2(k) = \tilde{a}_{02} - \tilde{a}_{12}(1 - l^{\tilde{a}_{22}k})$$

It is assumed that during multi-alternative optimization the value of variable z_m will be equal to 1 within the scope of probabilities change, p_{zm} $(1-\varepsilon, 1)$, and to 0 in the $(0, \varepsilon)$ area. Then, following [5]:

$$\tilde{\varphi}_1(k) - \tilde{\varphi}_2(k) = -\varepsilon(k)\lg\varepsilon(k) - (1-\varepsilon(k))\lg(1-\varepsilon(k)) + \varepsilon^k \lg(2^M - 1) = Q(k).$$

We set the value of function $\varphi_1(k)$ starting from the desired number of dominating optional alternative managerial decisions in the set $W_\partial - L^*$, define $\varphi_1^0 = \lg L^*$ and set the value of probability $P(\tilde{\varphi}_1(k) \le \varphi_1^0)$. Since it is known that the expectancy and variance of $\tilde{\varphi}_1(k)$ and $\tilde{\varphi}_2(k)$ depend on the expectancies of parameters $(\tilde{a}_0)$, $(\tilde{a}_1)$, $(\tilde{a}_2)$ respectively (a_{01}, a_{02}, a_{11}, a_{12}, a_{21}, a_{22} and variances $D(\tilde{a}_{21})$ and $D(\tilde{a}_{22})$), these values themselves follow the normal distribution law for fixed k. Then one may suggest the following algorithm to define k^0:

1) We find

$$Q^0 = \varepsilon^0 \lg\varepsilon^0 - (1-\varepsilon^0)\lg(1-\varepsilon^0) + \varepsilon^0 \lg(2^M - 1)$$

for a prescribed ε^0.

2) For L^* prescribed with respect to φ_1 the probability is

$$P(\tilde{\varphi}_1(k) \le \varphi_1^0) = P([\tilde{Q}_k - \tilde{\varphi}_2(k)] \le \varphi_1^0.$$

3) With the account of normal distribution of $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$ and the exponential form of retrieved relationship we proceed to the relationship for normalized distribution function Φ [10, 11]:

$$\Phi\left(\frac{\frac{1}{k^0}\ln\left(\frac{\varphi_2^0 - Q^0 - a_{02}^2 + a_{12}^2}{a_{22}^2}\right)}{D^{\frac{1}{2}}(\tilde{a}_{22})}\right) = \Phi\left(\frac{\frac{1}{k^0}\ln\left(\frac{\varphi_1^0 - a_{01}^1 + a_{11}^1}{a_{21}^2}\right)}{D^{\frac{1}{2}}(\tilde{a}_{21})}\right).$$

4) Finally we retrieve k^0 from that relationship.

Upon cutting off the iteration process for $k=k^0$, we follow the procedure outlined in Section 3.2 to form the set W_∂.

The structure of algorithmic procedure used to implement the virtual interaction mode is shown in figure 1.

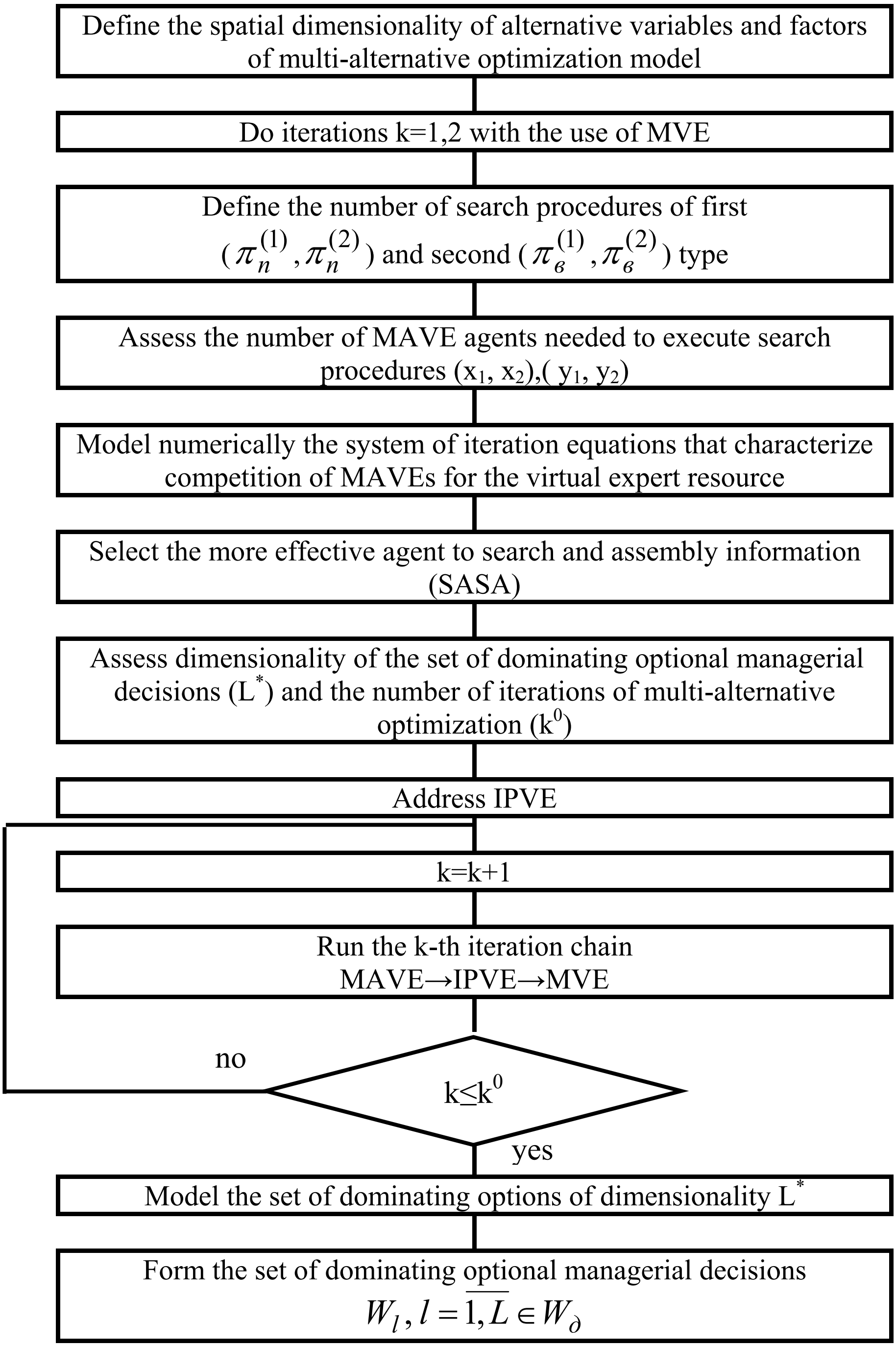

Figure 1. Structural flowchart of algorithmic procedure for the virtual interaction mode

2. Dual interaction mode

The principal task of developing the algorithm of dual mode interaction between the virtual and real experts (Section 2.2) in the environment of managerial decision making consists in forming a matrix game (MG) [11, 12]. The first player (A) is the multi-alternative virtual expert. Following Section 3.2 it forms the set of dominating options $W_l \in W_\partial, \quad l = \overline{1,L}$, which present strategies $A_l, \quad l = \overline{1,L}$. The second player (B) assesses these options according to each criterion $\psi_i, \quad i = \overline{1,I}$. It has two strategies: the first (strategy B_1) consists in using MVE for each criterion $\psi_i, \quad i = \overline{1,I}$ followed by definition of probabilities with the procedures of Section 3.2 and selection of l-th option $q_l^{\psi_i}, \quad l = \overline{1,L}$. The second strategy ($B_2$) consists in bringing in a real expert that offers subjective assessments of each option W_l following the criteria ψ_i. As for strategy B_1, we get probabilistic assessments of $q_l^{\psi_i}$, normalized over the interval [0, 1] immediately. For strategy B_2 it is suggested to use procedures from Section 3.1 to construct the model $\psi_i(x)$, where x is the vector of values of varied parameters, define the values $\psi_{il}(x_l)$ for $l = \overline{1,L}$ and find the values $\delta_l^{\psi_i}$, normalized over the interval [0, 1]

$$q_l^{\psi_i} = \frac{\psi_{il} - \min\limits_{l=\overline{1,L}}\{\psi_{il}\}}{\max\limits_{l=\overline{1,L}}\{\psi_{il}\} - \min\limits_{l=\overline{1,L}}\{\psi_{il}\}}$$

where $\max\limits_{l=\overline{1,L}}\{\psi_{il}\}$, $\min\limits_{l=\overline{1,L}}\{\psi_{il}\}$ are the maximum and minimum values of criterion ψ_i, respectively, calculated following the model over the set of options $W_l, \quad l = \overline{1,L}$. In the result we have a (2 x L) matrix game. The
matrix of such a game (2 x L) for the criterion Ψ_i, $i = \overline{1,I}$ has the form [5]:

B / A	B_1	B_2
A_1	$q_1^{\Psi_i}$	$\delta_1^{\Psi_i}$
:	:	:
A_l	$q_\ell^{\Psi_i}$	$\delta_\ell^{\Psi_i}$
:	:	:
A_L	$q_L^{\Psi_i}$	$\delta_L^{\Psi_i}$

To find the optimal strategy we use the procedure of reducing the game of the form $(2 \times L)^{\psi_i}$ to the game $(2 \times 2)^{\psi_i}$.

A \ B	B_1	B_2
W^1	$q^{1\Psi_i}$	$\delta^{1\Psi_i}$
W^2	$q^{2\Psi_i}$	$\delta^{2\Psi_i}$

where W^1, W^2 are the strategies of first and second players upon reduction to game $(2 \times 2)^{\psi_i}$.

The optimal probabilities and pure strategies for the (2 x 2) matrix are calculated following the formula [5]:

$$P(W^1) = \frac{\delta^{2\Psi_i} - q^{2\Psi_i}}{\left(q_1^{\Psi_i} + \delta^{2\Psi_i}\right) - \left(\delta^{1\Psi_i} - q^{2\Psi_i}\right)}, \quad P(W^2) = 1 - P(W^1).$$

A mixed strategy that consists of realizing pure strategies W^1 and W^2 randomly with probabilities $P(W^1)$ and $P(W^2)$ is optimal.

To proceed from game $(2 \times L)^{\psi_i}$ to game $(2 \times 2)^{\psi_i}$ it is suggested to:

1. Exclude strategies W_l that do not meet constraints imposed by the set of constraining functions φ (Section 2.1) from the matrix $(2 \times L)^{\psi_i}$.

2. Retain only the dominating strategies in the matrix, i.e. those on which both real and virtual experts express high confidence in their effectiveness.

3. Upon reducing preliminarily the number of pure strategies to $L_1 < L$ use the dichotomy principle. For that we find a mixed strategy $W_{1,2}^{\psi_i}$ from $(2 \times 2)^{\psi_i}$ matrices with strategies of the first player $A_1=W_1$, $A_2=W_2$

A \ B	B_1	B_2
W_1	$q_1^{\Psi_i}$	$\delta_1^{\Psi_i}$
W_2	$q_2^{\Psi_i}$	$\delta_2^{\Psi_i}$

and calculate assessed strategies of player B for the following probabilities of mixing pure strategies W_1 and W_2:

$$S_{1,2}^{\Psi_i} = \left|\begin{matrix} p^{\Psi_i}(W_1) \\ p^{\Psi_i}(W_2) \end{matrix}\right|, \; i=\overline{1, I}$$

The assessment is done as following:

$$\delta_{1,2}^{\Psi_i} = \delta_1^{\Psi_i} p^{\Psi_i}(W_1) + \delta_2^{\Psi_i} p^{\Psi_i}(W_2)$$

Next we find the mixed strategy $W_{1,2,3}^{\psi_i}$, $i = \overline{1,I}$ from matrices

A \ B	B_1	B_2
$W_{1,2}$	$q_{1,2}^{\Psi_i}$	$\delta_{1,2}^{\Psi_i}$
W_3	$q_3^{\Psi_i}$	$\delta_3^{\Psi_i}$

and repeat that transition to include strategy W_{L_3}.

In the result we find probabilistic assessments of mixed strategies for the criteria Ψ_i, $i = \overline{1,I}$:

$$S_{L_1-1,L_1}^{\Psi_i} = \left| \begin{matrix} p^{\Psi_i}(W_{1...L_1-1}) \\ p^{\Psi_i}(W_{L_1}) \end{matrix} \right|, \ i=\overline{1,I}$$

The structural flowchart of this dichotomy procedure is shown in figure 2.

Probabilities for pure strategies $p^{\Psi_i} W_\ell$ are used for the final selection. Preferred is the strategy with a maximum probability from among S^{ψ_i}, $i = \overline{1,I}$. In case it is a pure strategy, it is accepted as the best pure strategy $W_{MG}^{*\Psi_i}$. If it is a mixed strategy, we compare probabilities belonging to $S_{\Psi_i}^{L-2}$, $i = \overline{1,I}$ and proceed with the process until a pure strategy is finally selected.

Therefore, we have "*I*" best pure strategies for each criterion $W_{MИ}^{*\Psi_i}$, $i = \overline{1,I}$. The alternative W_l, corresponding to a pure strategy of the first player that has become the best for the highest number of *I* $(2 \text{ x } L)^{\Psi_i}$ matrix games is accepted as the agreed dual decision $W_{MИ}^*$.

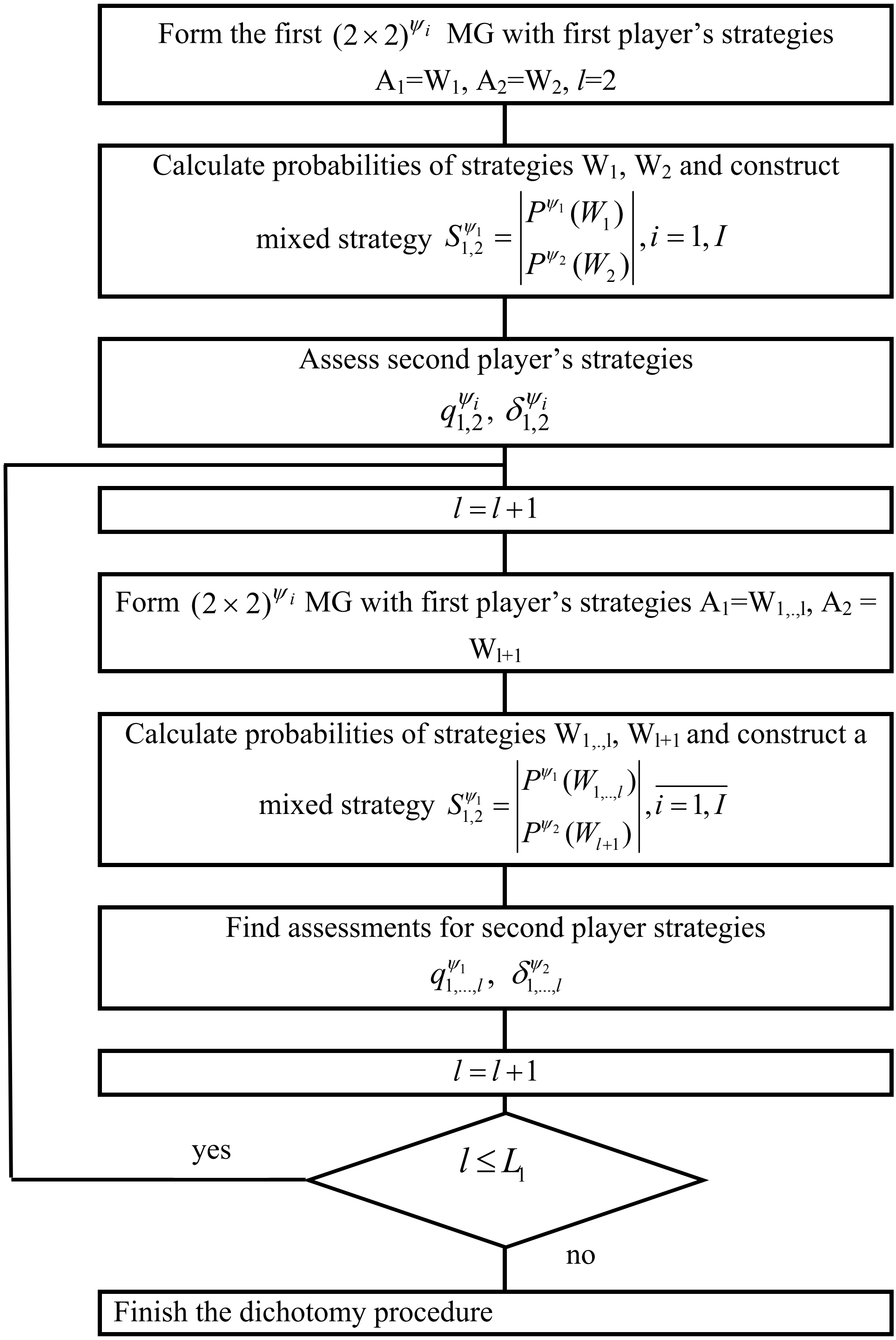

Figure 2. Structural flowchart of dichotomic transformation procedure for MG $(2\times L)^{\psi_i}$

Structural flowchart of the algorithm of dual mode interaction between the real and the virtual experts is shown in figure 3.

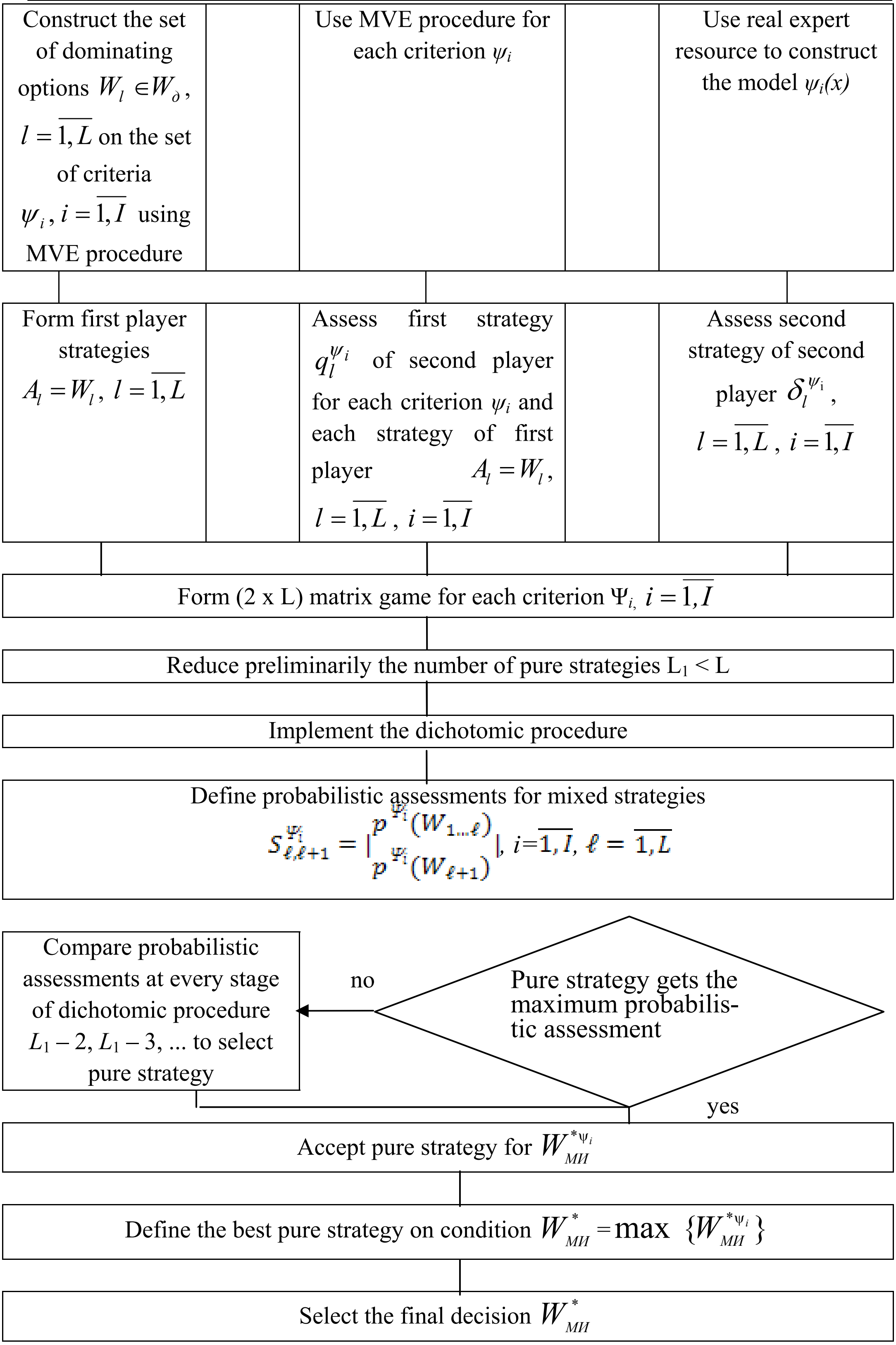

Figure 3. Structural flowchart of the algorithm of dual mode interaction between the virtual and real experts

3. Team interaction of experts of equal rank

Developing an algorithm for team interaction between the real and virtual experts to select optimal (rational) managerial decisions is based on man-machine procedure. It merges the resources of virtual and real experts and supports the dialogue with actual experiments via automatically offered questions, expert answers formalized next [5, 6, 13]. An iterative principle of constructing that procedure is suggested, immersing the question-answer process with the team of real experts into randomized environment. Such an environment provides for possible adaptive step-by-step tuning of distribution of random values that influence the training of experts during interaction. Experts may observe the outcome of decision they have taken on the previous step and adjust their expert valuation as needed, in dependence of answers by real experts and criteria Ψ_i, $i=\overline{1,I}$ assessed by the virtual expert. In case of MVE the set of dominating alternatives W_{∂} is formed in advance.

Assume a group of real experts numbered $d=\overline{1,D}$ assess the alternatives W_l, $l=\overline{1,L}$ according to criteria $\psi_i^{W_l}$, $i=\overline{1,I}$, their values defined by the virtual expert. To form a randomized environment we introduce the following random variables and distributions:

$\tilde{l}$ is a discrete random variable, its values $\tilde{l}=\overline{1,l}$ having the probabilities p_l, $l=\overline{1,L}$, $\sum_{l=1}^{L} p_l = 1$ that characterize the level of significance of alternative managerial decisions;

$\tilde{i}$ is a discrete random variable, its values $\tilde{i}=\overline{1,I}$ having the probabilities p_i, $i=\overline{1,I}$, $\sum_{i=1}^{I} p_i = 1$ that characterize the level of significance of criteria used to assess expert alternative decisions;

$\tilde{d}$ is a discrete random variable, its values $\tilde{d}=\overline{1,D}$, having the probabilities p_d, $d=\overline{1,D}$, $\sum_{d=1}^{D} p_d = 1$ that characterize the degree of trainability of real experts in the course of the dialogue and receival of new information on significance of alternative decisions.

The first step in this team procedure is to define the ranks of criteria using a priori ranging as outlined in Section 3.1.

In the result we obtain integer values of rank $r_i \in \overline{1, I}$ that decrease together with significance of criteria for the team of experts. It is suggested to use these values to obtain the initial distributions of discrete random values $\tilde{i}$ and $\tilde{d}$ so that the more significant criterion or better trained expert gains higher probability of being involved in the search in randomized environment:

$$p_i^1 = 1 - \frac{r_i}{\sum_{i=1}^{I} r_i}, \; i=\overline{1,I}$$

$$p_d^1 = \frac{\sum_{i=1}^{I}(r_i - r_i^d)^2}{\sum_{d=1}^{D}\sum_{i=1}^{I}(r_i - r_i^d)^2}, d = \overline{1,D}$$

where r_i^d is the rank attributed to *i*-th criterion by *d*-th expert,

$(r_i - r_i^d)^2$ is the degree of deviation of *d*-th expert rank from its average value.

Since no information on preferred alternatives $W_{i,}$ is available during the initial stage, a uniform distribution is assumed for the discrete random variable $\tilde{l}$:

$$p_l^1 = \frac{1}{L}, \; l = \overline{1,L}\cdot$$

Next we use normalized values of the criteria $\hat{\psi}_i = \frac{\psi_i - \psi_i^{\min}}{\psi_i^{\max} - \psi_i^{\min}}$, where $\psi_i^{\min}, \psi_i^{\max}$ are the respective minimum and maximum values of *i*-th criterion on the set of alternatives W_l, $l=\overline{1,L}$.

At each *k*-th iteration ($k > 1$, $k = 2, 3,..$) the following sequence of steps is taken:

1. Following the distribution p_d^k, $d = \overline{1,D}$ we generate the value of discrete random number $\tilde{d} = d^k$.

2. Following the distribution p_l^k, $l = \overline{1,L}$ we generate the value of discrete random number $\tilde{l} = l^k = j$ and present it to expert d^k to valuate the alternative W_j.

3. Man-machine procedure is realized as a dialogue with expert number d^k. He/she is asked: “The value of which of the criteria characterized by alternatives W_j, fails to meet it to the worst degree?”

Let the answer be: “Criterion number i^k”.

Next question is: “To what degree should the values of criteria $\Psi_{i,}$ $i = \overline{1,I}$ characterizing alternative W_j be changed to have a desired improvement”?

That degree is specified by a linguistic variable <should be changed> with its grades of <strongly> <significantly > <somewhat> <a little> <very little> given in [5].

4. The set of criteria numbered i_1^k, …, i_S^k is considered, the grade of their linguistic variable being <strongly>. This situation is formalized, first, as sign assessment:

$$\theta_i^k = \begin{cases} 1, & \text{if } i \in i_1^k, \ldots, i_S^k \\ -1, & \text{in the opposite case} \quad i = \overline{1, I} \end{cases}$$

and, second, as the average value of membership function μ [58] of the considered linguistic variable:

$$\mu^k = \frac{\sum_{i=i_1^k}^{i_S^k} \mu_i^k}{S} ,$$

where μ_i^k is the value of membership function for *i*-th criterion on *k*-th iteration.

It is suggested to use the function presented in [5] as the membership function.

5. Sets of criteria are considered, the grade of their linguistic variable being <very little>, and the number of such criteria T_j^k is calculated. In case we review all the alternatives W_l, $l = \overline{1, L}$ in the course of our random selection, the result will be the values T_l^k for all the alternatives $l = \overline{1, L}$.

6. We define the new distribution of the discrete random variable $\tilde{i}$ using the information obtained in the course of dialogue with the expert.

$$p_i^{k+1} = \frac{p_i^k + \frac{1}{S}\chi(\theta_i^k)\varepsilon^{k+1}}{1 + \varepsilon^{k+1}} , \quad i = \overline{1, I}$$

where ε^{k+1} is the step taken when calculating the probability p_i at (*k* + *1*)-th iteration.

$$\varepsilon^{k+1} = \varepsilon^k \exp\left\{ \frac{M^k}{S} \sum_{i=i_1^k}^{i_S^k} Sign\left[\theta_i^{k-1} \cdot \theta_i^k\right] \right\},$$

θ_i^{k-1} is the value of sign assessment of *i*-th criterion at (*k* – *1*)-th iteration,

$\chi(a)$ is the characteristic function,

$$\chi(a) = \begin{cases} 1, & \text{if } a > 0, \\ 0, & \text{if } a \le 0. \end{cases}$$

7. We do sign assessment of significance of alternatives W_l on condition $\sum_{i=1}^{I} p_i^{k+1} \hat{\psi}_i^{W_j} > \sum_{i=1}^{I} p_i^k \hat{\psi}_i^{W_j}$:

$$\theta_l^k = \begin{cases} 1, & \text{if } l = j \\ -1, & \text{in the opposite case}, \quad l = \overline{1, L} \end{cases}$$

8. We define the new distribution of discrete random variable $\tilde{l}$ in agreement with sign assessment on condition $\sum_{i=1}^{I} p_i^{k+1}\hat{\psi}_i^{W_j} > \sum_{i=1}^{I} p_i^{k}\hat{\psi}_i^{W_j}$

$$p_l^{k+1} = \frac{p_l^k + \chi(\theta_l^k)\gamma^{k+1}}{1+\gamma^{k+1}}, \ l = \overline{1, L},$$

while condition $\sum_{i=1}^{I} p_i^{k+1}\hat{\psi}_i^{W_j} \le \sum_{i=1}^{I} p_i^{k}\hat{\psi}_i^{W_j}$ yields

$$p_l^{k+1} = p_l^k, \ l = \overline{1, L}$$

where γ^{k+1} is the step taken to calculate the values of probabilities p_l during $(k + 1)$-th iteration,

$$\gamma^{k+1} = \gamma^k \exp\left\{\xi \sum_{l=l_1}^{V} Sign\left[\left(\sum_{i=1}^{I} p_i^k\hat{\psi}_i^{W_j} - \sum_{i=1}^{I} p_i^{k-1}\hat{\psi}_i^{W_j}\right)\left(\sum_{i=1}^{I} p_i^{k+1}\hat{\psi}_i^{W_j} - \sum_{i=1}^{I} p_i^{k}\hat{\psi}_i^{W_j}\right)\right]\right\},$$

$\xi > 0$ is the prescribed step size.

9. The distribution of discrete random variable $\tilde{d}$ remains unchanged:

$$p_d^k = p_d^1, \ d = \overline{1, D}$$

10. The above man-machine procedure is stopped after searching through all the alternatives W_l , $l = \overline{1, L}$.

11. The best option W^* is selected as following:

1) We define the sub-set of alternatives $L_1^* : \max_l T_l^k$;

2) We define the sub-set of alternatives $L_2^* : \max \sum_{i=1}^{I} p_i^k \hat{\psi}_i^{W_l}$;

3) We choose

$$W_i^* : \max_{l \in L_1^* \cap L_2^*} \sum_{i=1}^{I} p_i^k \hat{\psi}_i^{W_l}.$$

as the best alternative option.

Finally: $W^* = W_l^*$.

Now we proceed to explain adaptation inside the team of real and virtual experts in randomized environment. First, it becomes possible to assess in the dialogue mode the degree of satisfaction with the newly obtained values of criteria by a single expert for a single alternative at k-th step.

Following the distribution $p_d^1, d=\overline{1,D}$, it is particularly the experts who offer ranking assessments of significance of criteria closest to the averages that are involved more often in the dialogue, i.e. experts with most coherent assessments.

Distribution p_i , $i=\overline{1,I}$ is tuned for higher significance of probabilities of those criteria that most often show the strongest disagreement with experts agreed on various alternatives. Note that accounting for the algorithm of tuning p_l , $l=\overline{1,L}$ and selecting through all the alternatives, the probabilities of involving alternatives for expert assessment increase for higher average value of weighted convolution $\sum_{i=1}^{I} p_i^k \cdot \hat{\psi}_i^{W_l}$. Upon a certain number of iterations for the alternative that represents the best combination of values of criteria, for example, l_1, the probability p_{l_1} occupies a large part of the interval [0, 1], forcing the other alternatives out, since other probabilities become considerably smaller than p_{l_1}. In other words, experts have the alternative l_1 presented oftener for assessment and it is on that alternative that experts finally agree in their opinion on the significance of criteria and distribution p_i^k stabilizes. It is then said that p_i^k were obtained by adapting weight coefficients of criteria in the weighted average convolution

$$\sum_{i=1}^{I} \alpha_i \hat{\psi}_i, \; 0 \le \alpha_i \le 1, \; \sum_{i=1}^{I} \alpha_i = 1,$$

where $\alpha_i = p_i^k$, since $0 \le p_i^k \le 1$, $\sum_{i=1}^{I} p_i^k = 1$.

Choosing the size of the step for the second level ε^{k+2}, γ^{k+1} is organized so that, first, the value of membership function calculated for the specific grade of linguistic variable is used to tune probabilities p_i, and, second, the aftereffect of expert's answer on $(k-1)$-th iteration is accounted for to tune p_i and p_l. The latter makes it possible to change significantly the distribution of probabilities, provided expert assessments coincide on the $(k-1)$-th and k-th iterations; no sharp changes are introduced if there is no such coincidence.

The structural flowchart for the team mode selection algorithm in virtual expert environment is shown in figure 4.

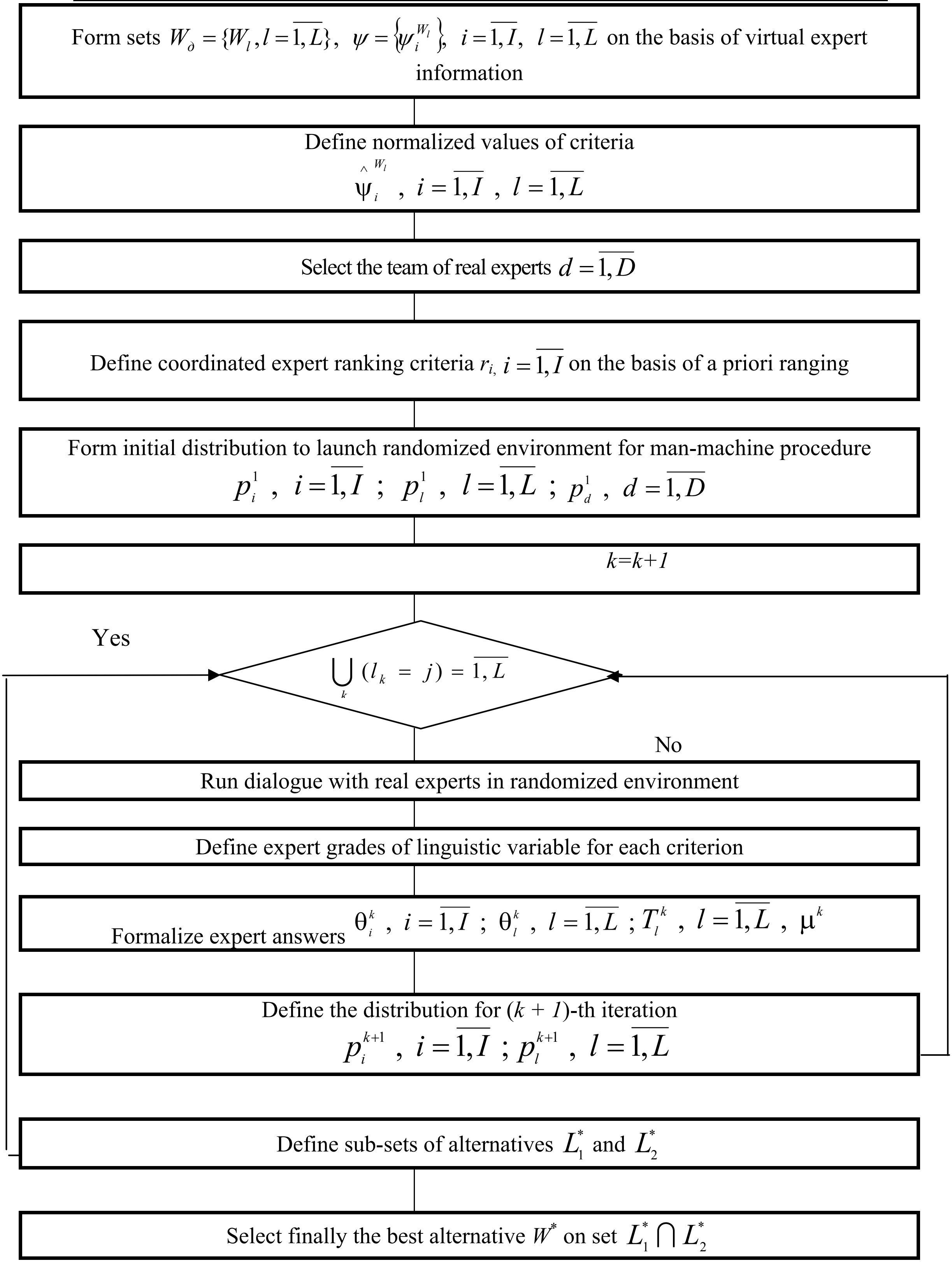

Figure 4. Team mode selection in virtual expert environment. Algorithm structural flowchart

4. Team interaction of experts of equal rank

Team mode with dominating expert is based on agreeing the assessments of priority of alternative optional managerial decisions by that expert with assessments by a team of peer experts [5, 6].

Assume that the dominating expert's number is $d = 1$, while other experts belong to $d = \overline{2, D}$. Going by their experience of interacting with a dominating expert (manager) and accounting for their intuition, logical analysis and experience these other experts use the question-answer procedure to assess the effectiveness of each alternative $W_l,\ l = \overline{1, L}$ proposed by expert #$d = 1$.

To organize the question-answer session we form a special linguistic structure. At the first stage we connect valuation of alternatives $W_l,\ l = \overline{1, L}$ by experts $d = \overline{2, D}$ with distributing all such alternatives into 3 classes that correspond to the effectiveness of the chosen decision:

- class A: the alternative W_l is effective with probability close to 1;
- class B: the alternative W_l is ineffective with probability close to 1;
- class C: the alternative W_l is effective with probability less than 0.5.

Expert #$d = 1$ presents his/her assessed allocation of alternatives $W_l,\ l = \overline{1, L}$ to classes A, B, C.

Valuation by experts $d = \overline{2, D}$ is based on the dialogue composed of answers to K questions of alternative form:

$$\alpha = (\alpha_1, ..., \alpha_k, ..., \alpha_K).$$

The task of constructing linguistic structure during the second stage consists in selecting such number of questions K that would make it possible to assess the degree of agreement (state β) between the dominating expert and each of $d = \overline{2, D}$ experts of equal rank. To retrieve the value of K we use the entropy approach [14-16].

With three classes available for the dominating expert and experts of equal rank to distribute alternatives to, state β has nine possible outcomes that remain equally probable prior to the dialogue procedure. Hence, the entropy $Э(\beta) = \log 9$. Meanwhile alternative questions that accept positive or negative answers only yield the entropy of the dialogue mode α:

$$Э(\alpha) \le K \log 2,$$

where $Э(\alpha) = \log 2$ for alternative answers.

Information obtained in the dialogue mode has to be at least as diverse as the outcomes for state β, which yields a relationship

$$Э(\alpha) \ge Э(\beta)$$

or

$$K \log 2 \geq \log 9.$$

Therefore, $K \geq \dfrac{\log 9}{\log 2}$, i.e. $K \geq 4$.

As for the third stage of constructing linguistic structure it is suggested to organize question-answer exchange as a sequence of four questions for each alternative $W_l,\ l = \overline{1, L}$:

1. Has expert $d = 1$ correctly rated alternative W_l to class A or B?

2. Has expert $d = 1$ correctly rated alternative W_l to class B?

3. Does expert of $d = \overline{2, D}$ believe alternative W_l to be effective with a probability higher than 0.5?

4. Has expert $d = 1$ correctly rated alternative W_l to class A?

Alternative answers (1,0) to the above questions yield $2^4 = 16$ situations. Meanwhile, placing alternative W_l in class A, B or C produces 9 situations only, i. e. 4 questions are redundant. However, having $2^3 = 8$ questions would prevent clearing all 9 situations. Table 4.1 below shows the decisions in dependence of answers to all the 4 questions.

Table 4.1. Adopted decisions

No.	**Question 1**	**Question 2**	**Question 3**	**Question 4**	**Adopted decision**
1	1	1	1	1	A
2	0	1	1	1	C
3	1	0	1	1	A
4	0	0	1	1	–
5	1	1	0	1	A
6	0	1	0	1	–
7	1	0	0	1	–
8	0	0	0	1	–
9	1	1	1	0	C
10	0	1	1	0	C
11	1	0	1	0	B
12	0	0	1	0	–
13	1	1	0	0	B
14	0	1	0	0	–
15	1	0	0	0	B
16	0	0	0	0	–

Answers by experts in the question-answer mode should be treated as voting on l-th alternative $l = \overline{2, L}$, the votes distributed into three classes A, B and C:

$$N_{A_j}^{A},\ N_j^{Б},\ N_j^{B}.$$

Following the majority rule (Γ31), the final decision according to expert valuation consists in putting it with the class of maximum N_{W_l}.

In case $N_{W_l} = N_{W_l}^{A}$, the alternative W_l contends for the optimal decision but needs coordination with the dominating expert. The final decision on the set of alternatives corresponding to that condition is taken by the dominating expert (Γ32).

Therefore, the question-answer procedure combined with the leading role of valuation by the dominating expert helps to choose the optimal decision W^* in team mode.

The dominating expert may agree with opinions by experts of equal rank and choose an optimal decision on condition:

$$W^* \to \max_l \{W_l : N_{W_l}\}$$

In other case the dominating expert may choose an alternative W_l ($N_{W_j} < N_{W^*}$) as his/her best decision and apply the algorithm placing W_l among the leaders additionally, provided the respective resource is available and it is possible to define the influence of that resource on winning the leading position [17-19].

The structural flowchart of team mode selection algorithm with dominating expert is shown in figure 5.

Conclusion

As a result of the undertaken study, a set of algorithms for interaction of components of an expert and virtual resource of a procedural type in optimal managerial decision making, is developed. The algorithm of virtual mode of interaction, based on the basic procedure of multialternative optimization; dual regime based on the matrix game development; collective mode of peer experts interaction, based on the iterative principle with immersion of question-answer process with a team of real experts in a randomized environment; collective mode with a dominant expert, based on the process of harmonizing its priority assessments with those of alternative managerial decision options provided by peer experts. The developed algorithms can be integrated into the single environment of a decision-making system and be used as a means of intellectual support.

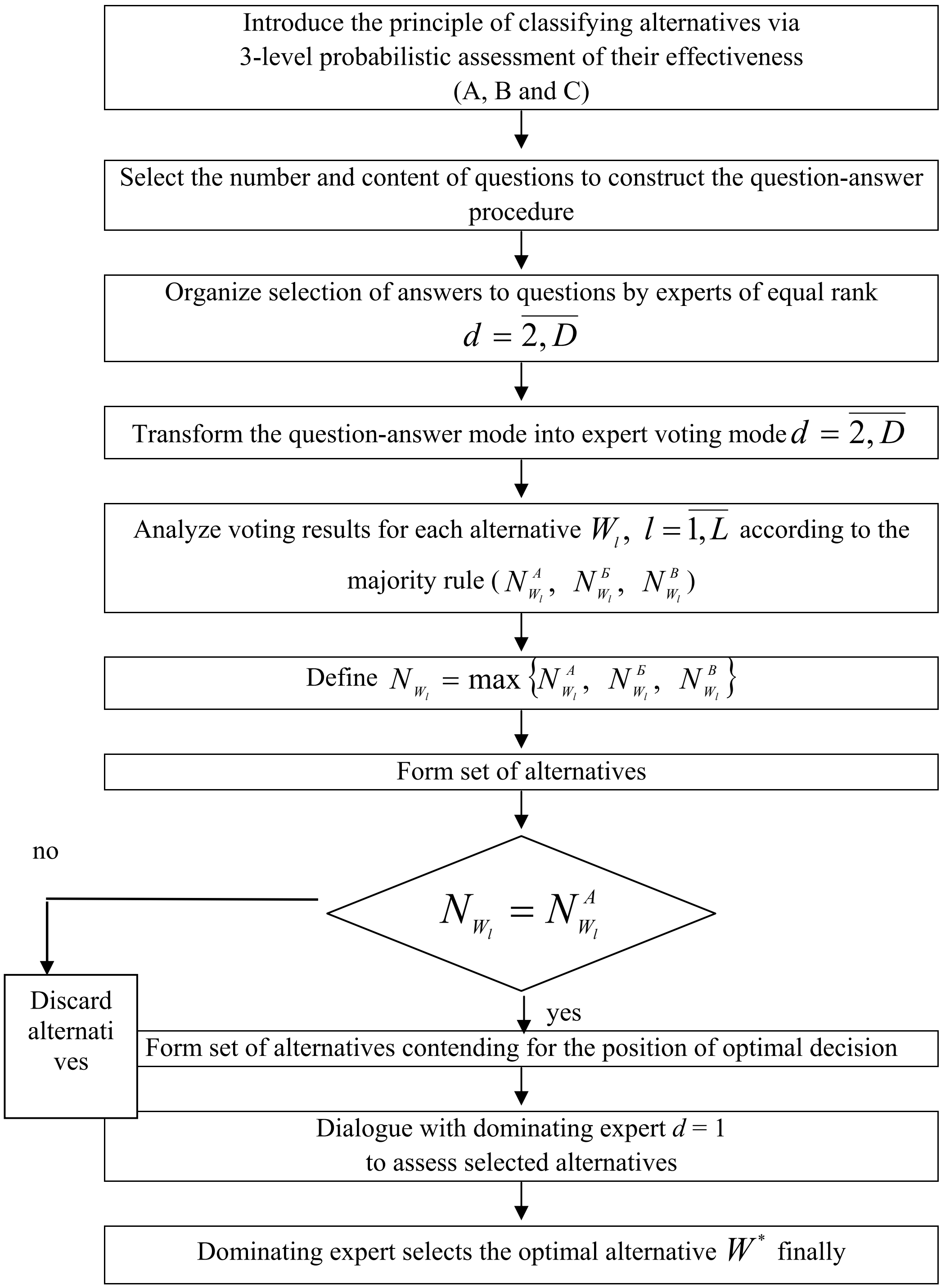

Figure 5. Team mode selection algorithm with dominating expert

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