Modeling of a numeracy of dominant options of multialternative optimization

Igor Lvovich, Juraj Štefanovič

Abstract:

One of the problems arising in management of large systems, is a selection of the best management options. To solve this problem an approach that allows to generate a numeracy of perspective options for managerial decisions is proposed. Development of a numeracy of dominant options of managerial decisions is based on the application of basic algorithmic procedures of variation modeling and building of multiple-optimization models. To use basic algorithmic procedures of variation modeling, the typical tasks are highlighted and their formalization with inclusion of alternative variables, allowing to develop a numeracy of perspective options through the variability of indicators, is performed. At that the problem of dichotomous reduction arises. Integration of all stages of decomposition and equivalent transition to analytical optimization models into a single scheme allowed to define a method of mathematical modeling of dichotomous reduction of complex systems diversity. To identify the dominant option, an algorithm that combines computational procedures for determination of the preference vector and the rational choice procedures, is developed.

Key words:

Simulation modeling, multialternative optimization, dichotomous reduction, rational choice, decision support.

ACM Computing Classification System:

Combinatorial algorithms, Algebraic algorithms, Nonalgebraic algorithms, Symbolic calculus algorithms, Exact arithmetic algorithms, Hybrid symbolic-numeric methods.

Introduction

In handling the problem of large systems management there is a problem of analysis, evaluation, comparison of alternative numeracy of options on a number of a variety of divergent criteria as a general principle. To formalize this problem, the theory of decision-making is used. However, the standard developed methods are not always effective in solving large-scale problems. It requires improvement of existing and development of new algorithms, including combined algorithms. In particular, one of the perspective areas of research is the use of variation modeling procedures and multialternative optimization techniques.

1. Formalization of typical tasks of development of a numeracy of perspective options

To use such algorithmic procedures of variational modeling based on multi-alternative optimization models one needs to identify typical problems and formalize them including alternative variables z_m that make it possible to form sets of prospective options according to factor variations.

Consider the basic class of problems $\overline{\beta_1,\ \beta_4}$ [1].

We substantiate the problems ($\overline{\beta_1,\ \beta_4}$) and calculate the respective a priori entropy $H(l, p^1) = \lg L$.

1. Problem β_1 consists in dividing the sets $w_g = \overline{1, W_g}$ into two groups, one of them forming sub-sets $\hat{W}_g \in W_g,$ that meet requirements $F_i^*, i = \overline{1, I}$. Such a division results in the following number of options available for selection:

$$L = 2^{Wg},\ g = \overline{1, G}. \tag{1.1}$$

The a priori entropy of each g-th problem is

$$H_g(\beta_1) = W_g \lg 2,\ g = \overline{1, G}. \tag{1.2}$$

2. Problem β_2 consists in selecting a single element in each sub-set $\overline{1, \hat{W}_g}$, $g = \overline{1, G}$ following the requirements $F_i^*, i = \overline{1, I}$. The number of options for selection is then

$$L = \prod_{g=1}^{G} \hat{W}_g\ ;$$

the respective a priori entropy being

$$H(\beta_2) = \sum_{g=1}^{G} \lg \hat{W}_g\ . \tag{1.3}$$

3. Problem β_3 consists in ranging options W_g according to index g, i.e. in selecting the order of antecedence of elements $w_g < w_t$; $g \neq t$; $g,\ t \in \overline{1, T}$; sign “<” indicates that element t follows after element g. The problem considered is related to either defining the position of element w in the sequence $\overline{1, T}$, or to defining the order of proceeding from the initial element number $t = 0$ to the following elements numbered $t > 0$. The number of options for selection is $L = T$!

To assess the value of a priori entropy for any integer $T \geq 6$ we use the inequality [1]:

$$\left(\frac{T}{2}\right)^T \rangle T! \rangle \left(\frac{T}{3}\right)^T,$$

then $T \lg \frac{T}{2} \rangle H(\beta_3) \rangle T \lg \frac{T}{3}$.

4. Problem β_4 consists in merging together elements with numbers $g = \overline{1,G}$ into the $g' = \overline{1,G'}$ group. The number of options for it is:

$$L = (G')^G,$$

and the a priori entropy is:

$$H(\beta_4) = G \lg G'. \tag{1.4}$$

To assess the entropy of dichotomic optimization model we assume that it coincides with the entropy of test A corresponding to Boolean structures (μ), i. e.

$$H(\mu) = H(A).$$

We now calculate the value of entropy for the disjunctive test $A_m = (m = \overline{1,M})$:

$$H(A_m) \leq \lg N_m,$$

where the equality sign corresponds to equal probabilities of results $P(A_{mn}) = \frac{1}{N_m}$.

The entropy of a complex test A:

$$H(A) = H(A_1 \ldots A_m \ldots A_M) \leq H(A_1) + \ldots + H(A_m) + \ldots + H(A_M) \leq \sum_{m=1}^{M} \lg N_m.$$

If $N_1 = \ldots = N_m = \ldots = N_M = N$, then $H(A) \leq M \lg N$.

In case $N_m = 2 \forall m = \overline{1,M}$

$H(A) \leq M \lg 2$.

2. Conditions of dichotomous optimization model adequacy and set-theoretical statement of a problem

Thus we have computational formulas to assess the a priori entropy for problems $(\overline{\beta_1, \beta_4})$ and the entropy of Boolean structures within the scope of model μ. We now formulate the conditions of correspondence between these values. We shall call them conditions for adequacy of dichotomic optimization model and theoretical multiple setting of the problem.

The dichotomic optimization model (μ) describes the problem (β) adequately in case the outcome of test A determines fully the outcome of problem (β).

Theorem: For dichotomic optimization model to describe adequately problem (β) it is necessary and sufficient to meet the following condition:

$$H(\beta) \leq H(\mu). \tag{2.1}$$

Let us prove its necessity. Test A taken according to model (μ) makes it possible to obtain information on problem (β):

$$I(A, \beta) = I(\mu, \beta) = H(\beta) - H_\beta(\mu),$$

where $H_\mu(\beta)$ is the entropy of problem (β) in case test A is taken following model (μ).

On the other hand:

$$I(\beta, \mu) = H(\mu) - H_\beta(\mu),$$

where $H_\beta(\mu)$ is the entropy of test A corresponding to the respective model (μ) for the outcome of problem (β).

Since

$$I(\mu, \beta) = I(\beta, \mu),$$

we have

$$H(\beta) - H_\mu(\beta) = H(\mu) - H_\beta(\mu). \quad (2.2)$$

In case condition (2.1) is met, we may proceed from (6) to inequality

$$H_\mu(\beta) \le H_\beta(\mu). \quad (2.3)$$

Since $H_\beta(\mu)$ may be equal to 0, inequality (2.3) is only satisfied in case $H_\mu(\beta)=0$. It means that the outcome of test A defines fully the outcome of problem (β) and dichotomic optimization model (μ) describes problem (β) adequately.

Now we proceed to prove sufficiency. If model (μ) describes the problem (β) adequately then $H_\mu(\beta) = 0$. It follows from (2.2) then:

$$H(\beta) = H(\mu) - H_\beta(\mu).$$

Finally we have

$$H(\beta) \le H(\mu),$$

Q. E. D.

This theorem makes it possible to find dimensionality and structure of dichotomic optimization model (μ) for problems ($\overline{\beta_1, \beta_4}$).

Rule 1. Dimensionality of dichotomic optimization model (μ_1) adequate to problem (β_1) is:

$$M = W_g \, (g = \overline{1, G}) \quad (2.4)$$

A model inducting the test for $N_m = 2$, $\forall m = \overline{1, M}$ corresponds to problem (β_1). Then it follows from condition (2.1) with the account of (1.2) that
$H_j(\beta_1) = W_g \lg 2 \le H_j(\mu_1) \le M \lg 2$, $M=W_g$ $(g = \overline{1, G})$.

Following (2.4) we introduce Boolean variables of multi-alternative optimization model:

$$W_g^1 = \begin{cases} 1, & \text{in case element is included in the ensemble} \quad \hat{W} \in W, \\ 0, & \text{in the opposite case} \end{cases}$$

$$(w_g = \overline{1, \mathrm{W_g}}).$$

Next we find the structure of objective function and model constraints for the task of forming the admissible set $\hat{W} \in W$. For that purpose we construct the matrix $a = \left\| a_{wg} \right\|$, assuming that

$$a_{wg} = \begin{cases} 1, & \text{in case element } \mathrm{w_g} \text{ meets the choice for the i-th factor element,} \\ 0, & \text{in the opposite case} \end{cases},$$

$$(w_g = \overline{1, \mathrm{W_g}}, \mathrm{i} = \overline{1, \mathrm{I}}).$$

In case one needs to find the minimum set $\overline{1, \hat{\mathrm{W}}_g}$, optimization model is presented as the problem of minimum coverage [2]:

$$\sum_{w_g}^{W_g} x_{wg}^1 \to \min,$$

$$\sum_{w_g=1}^{W_g} a_{iwg} x_{wg}^1 \geq 1, (\mathrm{i} = \overline{1, \mathrm{I}}),$$

$$x_{wg}^1 = \begin{cases} 1, \\ 0, \end{cases} (w_g = \overline{1, \mathrm{W_g}}).$$

In certain situations the available information is insufficient to construct the matrix $\|a_{iwg}\|$. One may only indicate the "value" a_{rj} of element w_g with respect to factor $F_i (i \in I_1)$, in other words – set the constraint $C_i (i \in I_2)$ with the account of "weight" characteristics c_{iwg} of separate elements. Then the optimization model is reduced to a multidimensional knapsack problem [2]:

$$\sum_{w_g=1}^{W_g} a_{wg} x_{wg}^1 \to \max,$$

$$\sum_{w_g=1}^{W_g} c_{iwg} x_{wg}^1 \leq C_{wg}, (\mathrm{i} = \overline{1, \mathrm{I_2}}),$$

$$x_{wg}^1 = \begin{cases} 1, \\ 0, \end{cases} (\mathrm{w_g} = \overline{1, \mathrm{W_g}}).$$

Rule 2. Dimensionality of optimization models (μ_2), adequate to problem (β_2) is:

a) $M = G, \mathrm{N_m^R} = \hat{W}_g (m = \overline{1, \mathrm{G}}),$ (2.5)

b) $M = \sum_{g=1}^{G} \frac{\lg \hat{W}_g}{\lg 2},$ (2.6)

$N_m = 2$, $\forall m = \overline{1, M}$.

In the first case we assume that model (μ_2) inducts a complex test A with its entropy meeting the general condition. Accounting for (1.3) and (2.1) we have then:

$$H(\beta_2) = \sum_{g=1}^{G} \lg \hat{W}_g \leq H(\mu_2) \leq \sum_{m=1}^{M} \lg N_m.$$

Expression (2.5) follows unambiguously then.

Following (2.5) we introduce Boolean variables for the dichotomic optimization model. Then

$$x_{wg} = \begin{cases} 1, \ \mathit{if} \quad w_g \in \hat{W} \quad \mathit{provides\ for\ meeting\ the\ conditions} \quad , i = \overline{1, I}, \\ \qquad 0, \ \mathit{in\ the\ opposite\ case.} \end{cases}$$

$$(w_g = \overline{1, \hat{\mathrm{W}}_\mathrm{g}}, g = \overline{1, G}).$$

Since complex test A is disjunctive over index n, the set of Boolean variables has to meet the following constraints for a given m:

$$\sum_{w_g}^{\hat{W}_g} x_{wg} = 1, \; (g = \overline{1, G})$$

In the second case we assume that model (μ₂) indicates a complex test A for $N_m = 2$, $\forall m = \overline{1, M}$. With the account of (10) it follows from condition (2.1) then

$$H(\beta) = \sum_{g=1}^{G} \lg \hat{W}_g \leq H(\mu_2) \leq M \lg 2 \cdot$$

And relation (2.6) follows from it.

To introduce Boolean variables corresponding to (2.6), one needs to present elements of the set $\hat{W}$ in their binary denomination. E. g., we have for $\hat{W}_g \leq 8, \forall g = \overline{1, G}$:

$$w_1 = x_1 + 2x_2 + 4x_3,$$
$$\vdots$$
$$w_G = x_{M-2} + 2x_{M-1} + 4x_M,$$

where

$$x_m = \begin{cases} 1, \\ 0, \end{cases} (m = \overline{1, M}).$$

Finally optimization models (μ_2) adequate to problem (β_2) are presented in the form of multi-criteria optimizing problems with Boolean and continuous variables. For case (a): $\Psi_i(x_{wg}) \to \max, (i \in I_1)$,

For case (b):

$$\Psi_i(x_m) \to \max, \; (i \in I_1),$$
$$x_m = \begin{cases} 1, \\ 0, \end{cases} (m = \overline{1, M}).$$

Since numbers $\hat{W}_g$ coincide with values 2^k ($k = 0, 1, 2, \ldots$), numbers of fictitious w_Φ correspond to certain sets of x_m. For them we introduce such functions $\varphi_i(w_\Phi)$, for which the requirement $F_i^*, i = \overline{1, I}$ is deliberately not met.

Rule 3. Dimensionality of dichotomic optimization models (μ_3) adequate to problem (β_3) is:

$$M = T, \; N \geq \frac{T}{2}.$$

Model (μ_3) inducts a complex test A, its entropy meeting the general condition. Then with the account of (2.1) we have:

$$T \lg \frac{T}{2} > H(\beta_3) < H(\mu_3) \leq M \lg N$$

That inequality is met for $M = T, \; N \geq \frac{T}{2}$, in particular, for $N = T$.

As indicated above, the order of antecedence of elements merged into a complex system is set in two ways:

a) an ordered sequence of numbers is set from the list of numbers $S - n = \overline{1,T}$ and each number has its corresponding m-th $(m = \overline{1,T})$ element from the list (s);

b) the initial element ($m = 0$) is specified, and one needs to find an optimal route for transition from element to element belonging to list s_l, eventually returning to initial elements.

We introduce Boolean variables for the first case:

$$x_{mn} = \begin{cases} 1, & \text{if element } w_n^* \text{ is attributed its m-th number in list } s_l \\ 0, & \text{in the opposite case} \end{cases}$$

$$(m = \overline{1,T}, \mathrm{n} = \overline{1,\mathrm{T}}).$$

Each option of the sequence has a corresponding value of some particular factor for the system $F = \Psi(x_{mn})$. In the result we have a dichotomic optimization model of the type of Assignment Problem [2, 3]:

$$F = \Psi(x_{mn}) \rightarrow extr,$$

$$\sum_{m=1}^{T} x_{mn} = 1, \quad (\mathrm{n} = \overline{1,\mathrm{T}}),$$

$$\sum_{n=1}^{T} x_{mn} = 1, \quad (\mathrm{m} = \overline{1,\mathrm{T}})$$

$$x_{mn} = \begin{cases} 1, \\ 0, \end{cases} (m = \overline{1,T},\ n = \overline{1,T}).$$

In the second case we associate Boolean variables with alternatives to the transition from m-th to n-th element:

$$x_{mn} = \begin{cases} 1, & \textit{in case the antecedence relation is met } Vm < Vn, \\ 0, & \textit{in the opposite case} \end{cases}$$

$$(m = \overline{0,T},\ n = \overline{0,T}).$$

Certain values of a particular factor $f = f(x_{mn})\,(m = \overline{0,T},\ n = \overline{0,T})$, the factor $F = \Psi(f(x_{mn}))$ related to them, correspond to transitions from one element to the other. Then the optimization model acquires the structure typical for the Travelling Salesman's Problem [2-4]:

$$F = \Psi(f(x_{mn})) \rightarrow extr,$$

$$\sum_{m=0}^{T} x_{mn} = 1, \quad (\mathrm{n} = \overline{1,\mathrm{T}}),$$

$$\sum_{n=0}^{T} x_{mn} = 1, \quad (\mathrm{m} = \overline{1,\mathrm{T}})$$

$$\pi_m - \pi_n + T x_{mn} \le T\text{-}1,$$

$$(m = \overline{1,T},\ n = \overline{1,T},\ m \ne n),$$

$$x_{mn} = \begin{cases} 1, \\ 0, \end{cases} (m = \overline{0,T},\ n = \overline{0,T}),$$

where π_m, π_n are arbitrary real values.

Rule 4. Dimensionality of dichotomic optimization models (μ_4), adequate to problem (β_4):

$$M = G,\ N \ge G'.$$

Model (μ_4) inducts a complex test (A), its entropy meeting the general condition. Then with the account of (4) we have:

$$H(\beta_4) = G \lg G' \le H(\mu_4) \le M \lg N .$$

That inequality is met for $M = G,\ N \ge G'$.

We introduce Boolean variables for model (μ_4):

$$x_{mn} = \begin{cases} 1, & \text{in case the } m\text{-th element is attributed to the } \mathbf{n}\text{-th group} \\ 0, & \text{in the opposite case.} \end{cases}$$

$$(m = \overline{1,G},\ n = \overline{1,G'}).$$

The optimal division into groups is characterized by some particular factor F that is a function of parameters $f(V_m)$ and variables x_{mn}. Note that in model (μ_4) each element W_m may only belong to a single group. Besides, there exists an antecedence order of elements $(V_g < V_m,\ g = \overline{1,G},\ m = \overline{1,G},\ g \ne m)$ [5-7]. Finally we arrive at the following multi-alternative optimization model:

$$F = \Psi(f(V_m), x_{mn}) \to extr ,$$

$$\sum_{n=1}^{G'} x_{mn} = 1,\ (m = \overline{1,G}),$$

$$V_g < V_m, (g = \overline{1,G},\ m = \overline{1,G},\ g \ne n ,$$

$$x_{mn} = \begin{cases} 1, \\ 0, \end{cases} (m = \overline{1,G},\ n = \overline{1,G'}).$$

Therefore, problems of multi-alternative aggregation that belong to class (β) are brought in correspondence with their adequate analytical optimization models.

3. Method of mathematical modeling of dichotomous reduction of complex systems diversity

Merging all the stages of decomposition and equivalent transition to analytical optimization models into a common scheme makes it possible to formulate a technique for mathematical modeling of dichotomic reduction of diverse complex systems that includes the following procedures:

1. Define the composition of local problems (β): decomposition.
2. Assess the a priori entropy of problems (β): $H(\beta)$.
3. Shape preliminarily the dichotomic optimization model (μ).
4. Assess the entropy $H(\mu)$ of a complex test corresponding to that dichotomic optimization model.
5. Assess the adequacy condition.
6. Set the final structure and dimensionality of the dichotomic optimization model following Rules 1 – 4.

The links between those stages are shown in the structural flowchart (fig. 1). A special feature of solving numerically the above problems of multi-alternative optimization consists in that all their listed types (free of constraints, having algorithmically described constraints or constraints of general type and multi-criteria problems) are solved within the scope of a single scheme.

It is implemented via the following set of algorithmic procedures [1]:

θ1 – generate admissible problem solution;
θ2 – tune the distribution laws for alternative variables;
θ3 – form computed prognostic assessments (V_{rasc});
θ4 – define the order of setting disjunctive tests;
θ5 – form prognostic expert assessments (V_{exp});
θ6 – account for general constraints and multi-criteriality.

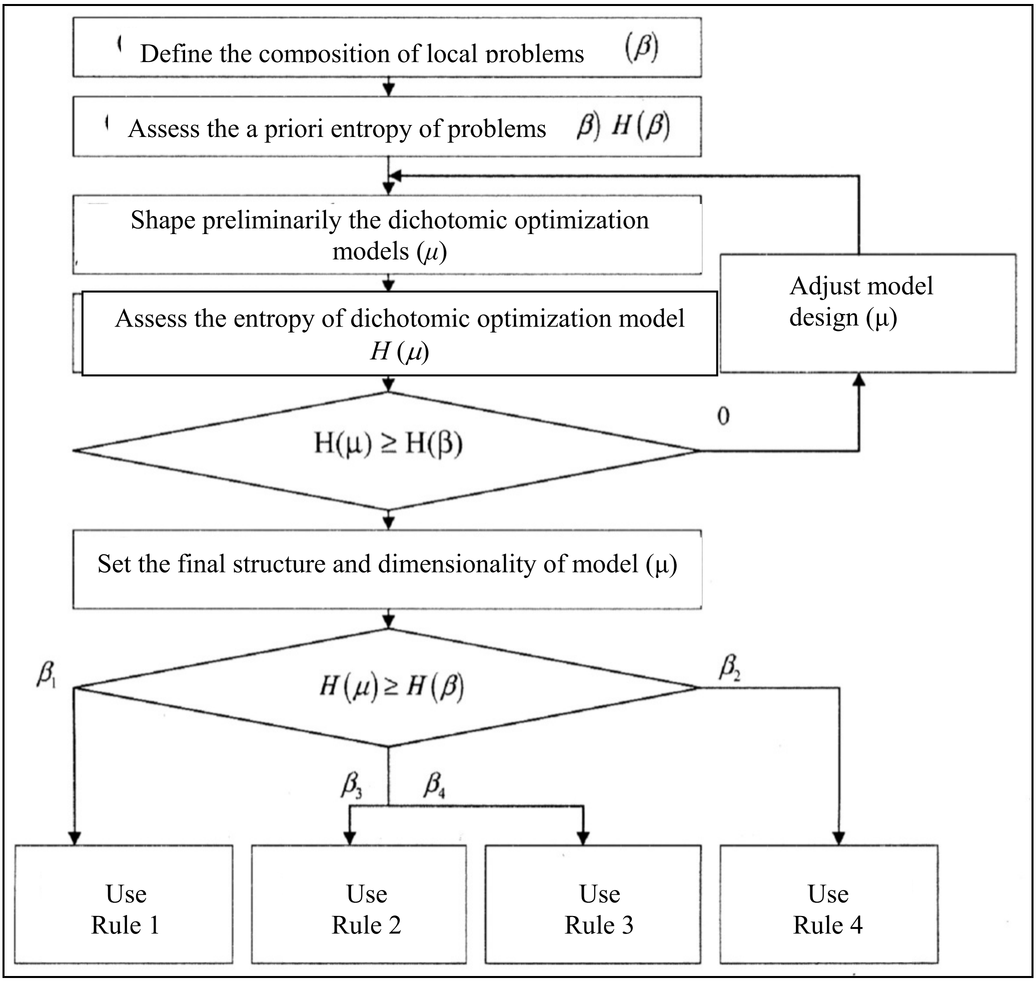

Figure 1. Structural flowchart of mathematical modeling of the process of dichotomic reduction

4. Rational choice method on a numeracy of complex systems ranked options

Algorithmic procedures ($\overline{\theta_1,\theta_6}$) make it possible to search through the set of alternative variables of optimization models. We demonstrate the possible organization of rational selection on the set of ranged options of complex systems that we shall call identifying the dominating option. We interpret the process of transiting from one optional solution to another, related to algorithmic procedures of multi-alternative optimization, as taking a chance path that corresponds to some finite Markov chain [2] with its variable transition matrix:

$$p^k = \left\| p_{l\omega}^l \right\|, (l = \overline{1,L}, \omega = \overline{1,L}, k = 1,2,...). \tag{4.1}$$

Consider some of its properties that make it possible to proceed from the distribution of random Boolean variables to the vector of preferential options.

In case of a multi-alternative optimization model of dimensionality $(M, N = 2)$ and algorithmic procedures (θ_3), (θ_4), we may proceed from a certain situation with its vector $x^l = \{x_m^l\}$, characterizing option $l \in \overline{1,L}$, to situation M belonging to $(L-1)$ others, or remain in the initial situation. Then elements of the transition matrix (4.1) are calculated as following:

$$p_{l\omega}^k = \sum_{M-1}^{m} |x_{m\omega} - x_{ml}| p^k (x_m = x_{m\omega}) p_m^k + \aleph_1 (|x_{m\omega} - x_{ml}|) p^k (x_m = x_{m\omega}) p_m^k \times$$
$$\times \left[\aleph_2 \left(\sum_{M-1}^{m} |x_{m\omega} - x_{ml}| \right) \right], (l \neq \omega, l, \omega \in \overline{1,L}), \; p_{ll}^k = \sum_{M-1}^{m} p^k (x_m = x_{ml}) p_m^k, l = \overline{1,L}, \tag{4.2}$$

where $p^k (x_m = x_{m\omega})$ is the probability for Boolean variable x_m to obtain its value $x_{m\omega}$, corresponding to the ω-th situation;

$$\aleph_1(x) = \begin{cases} 1, & \text{if} \quad x = 0 \\ 0, & \text{if} \quad x = 1 \end{cases} \qquad \aleph_2(x) \begin{cases} 1, & \text{if} \quad x = 0 \\ 0, & \text{if} \quad x \neq 1 \end{cases}$$

In case dimensionality of the dichotomic optimization model is (M, N) and there is a constraint $\sum_{n=1}^{N} x_{mn} = 1$, then, according to (θ_4) the process of taking the chance path should be considered separately for each vector $x_m = (x_{m1},...,x_{mn},...,x_{MN})$. One may proceed then from situation with its vector $x_m = (x_{m1} = 0,...,x_{mn} = 1,...,x_{MN} = 0)$ to $(N - 1)$ other situations or remain in the initial situation. The elements of transition matrix (4.1) are calculated as:

$$p_{l\omega}^k = p_{xmn}^k \tag{4.3}$$

Let us demonstrate the regularity of Markov chain with elements (12) of the transition matrix. It is known [1] that the transition matrix is regular when all matrix elements $(P_k)^\alpha$ are different from 0 for some integer $\alpha > 0$.

The number α means the number of steps needed to transit from one state to the other. According to (4.2) for $\alpha = 1$ one may transit from l-th situation to $(M + 1)$ other situations at a probability different from zero, i. e. the number of elements $p_{l\omega}^k$, different from zero in each line and each column of matrix p^k is equal to $(M + 1)$. The minimum number of steps over which the transition from l-th situation to $(L - 1)$ other situations is possible with non-zero probability is $\alpha = M$, and the transition matrix with elements (4.2) put to the power of $\alpha = M$ contains no elements equal to 0. Hence, it is regular, and the respective Markov chain is also regular.

Elements (4.3) of the transition matrix are not equal to zero for $\alpha = 1$, hence in case of dichotomic optimization model and (M, N) algorithmic procedures they generate a regular Markov chain.

For each k-th iteration the properties of a regular Markov chain [2] make it possible to retrieve analytically the components of vector $q = (q_1, \ldots, q_l, \ldots, q_L)$ on the basis of transition matrix. We start with its following properties:

1. Transition matrix put to the power of α ($(P)^{\alpha}$) tends to the probability matrix T for $\alpha \to \infty$.
2. Each line of matrix T presents one and the same probability vector $t = (t_1, t_2 \ldots, t_l, \ldots, t_L)$, all its components positive while $\sum_{l=1}^{L} t_l = 1$.
3. The rate of convergence of $(P)^{\alpha}$ to its limit T is exponentially fast, and matrices $(P)^{\alpha}$ and T remain close to each other for relatively small values of α.
4. Vector t is the only vector for each $tP = t$.
5. For any initial distribution of options

$$m(u_l^{\alpha}) \to t_l \text{ for } \alpha \to \infty,$$

where u_l^{α} is the share of time spent by the Markov process in state $l = \overline{1, L}$ (over the first α steps).

The first four properties are used to calculate the coordinates of vector $t = \{t_1, \ldots, t_l, \ldots, t_L\}$ by putting matrix P^k to the power of $\alpha > M$ or by solving the system of linear equations $tP = t$. The last property makes it possible to equate the values of vector q^k coordinates to the coordinates of vector t, i.e.

$$q_l^k = t_l. \tag{4.4}$$

Let us demonstrate that probabilities calculated after (4.4) are truly the quantitative assessments of selection probabilities which meet certain conditions of rational selection.

The above Markov chain with its set of states $s_l, (l = \overline{1, L})$ and transition probabilities $p_{l\omega}, l = \overline{1, L}, \omega = \overline{1, L}$ features the necessary properties [8] making it possible to look at values q_l^k as a quantitative assessment of selection probabilities $P(s_l : S)$ that meet the principle of sequential narrowing. Besides, these assessments agree with the theory of probabilistic ranging [1, 8], i.e., they not only characterize the probability of selecting option s_l from set S, but that of selecting it in the first place.

Following that reasoning, the complex system S_{ω} optimal on set S is such that

$$P(s_{\omega} : S) = \max\left\{ P\left(\underset{l=\overline{1,L}}{s_{\omega}} : S \right) \right\} \tag{4.5}$$

However, dealing with a multi-alternative optimization model μ that serves as the basis for automatic search procedures, it often appears impossible to formalize all the constraints related to synthesizing a complex system. To account for non-formalized constraints, a DMP is involved. Probabilities q_l are treated as probabilistic ranks then. It is suggested for decision-maker to consider some group L^* of highest ranks instead of a single option corresponding to condition (4.5). Rational selection is once again implemented following the principle of consequential narrowing on set $l = \overline{1, L}$, but now it goes with the account of qualitative factors that remained non-formalized in model μ.

Distribution (4.4) is used to identify the dominating option while achieving prescribed quality against a certain condition. The entropy function entering such a condition is averaged. Therefore, the values $H\left(l, q^k\right)$ may appear equal to each other for different distributions q^k (fig. 2).

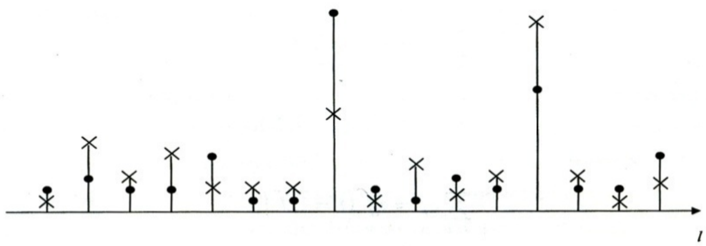

Figure 2. Distribution q of probabilities to select options of a complex system.
(• is the distribution $q^{k+\aleph} = \eta^1$; × is the distribution $q^{k+(\aleph-1)} = \eta^2$)

One should recourse to the procedures of rational selection when the series of $\aleph(\aleph = 1,2,...)$ iterations of distribution $q^{k+\aleph}$ are identical to each other. To assess such a situation, consider the formal description of preference using fuzzy relations [1].

Consider the distribution $\eta^1 = q^{k+\aleph}(\aleph = 1, 2,...)$ for $\eta^2 = q^{k+(\aleph-1)}$. How does one estimate quantitatively whether η^2 is not in preference to η^1 using the value:

$$\alpha_{\eta^1 \sim \eta^2} = 1 - (\alpha_{\eta^1 \eta^2} + \alpha_{\eta^2 \eta^1}),$$

where $\alpha_{\eta^1 \eta^2}$ is the degree of strict preference of η^1 over η^2;

$\alpha_{\eta^2 \eta^1}$ is the degree of strict preference of η^2 over η^1.

If distributions $\eta^1 = \left(\eta_1^1, ..., \eta_l^1, ..., \eta_L^1\right)$ and $\eta^2 = \left(\eta_1^2, ..., \eta_l^2, ..., \eta_L^2\right)$ are known, calculating the values of α follows the formulas:

$$\alpha_{\eta^1 \sim \eta^2} = \sum_{l=1}^{L} \varsigma_{ll}, \quad \alpha_{\eta^1 \eta^2} = \sum_{l<t} \varsigma_{tl}, \quad \alpha_{\eta^2 \eta^1} = \sum_{l>t} \varsigma_{lt}$$

where $\varsigma_{lt}, \varsigma_{tl} (l = \overline{1, L}, \ t = \overline{1, L})$ are solutions for the problem of linear programming:

$$\sum_{l=1}^{L} \sum_{t=1}^{L} \varsigma_{lt} \left| u(l_l) - u(l_t) \right| \to \min, \quad \sum_{t=1}^{L} \varsigma_{lt} = \eta_l^1, \quad (l = \overline{1, L}), \quad \sum_{t=1}^{L} \varsigma_{lt} = \eta_l^2, \quad (t = \overline{1, L}),$$

where $u(l_l)$, $u(l_t)$ are arbitrary functions that retain their order along a discrete scale $\overline{1, L}$.

Fuzzy relation $\alpha_{\eta^1 \sim \eta^2}$ makes it possible to study such preference situations as:

– strict preference: $\alpha_{\eta^1\eta^2} = 1,\ \ \alpha_{\eta^2\eta^1} = \alpha_{\eta^1 \sim \eta^2} = 0$;

– indifference: $\alpha_{\eta^1 \sim \eta^2} = 1,\ \ \alpha_{\eta^1\eta^2} = \alpha_{\eta^2\eta^1} = 0$;

– large preference: $\alpha_{\eta^1\eta^2} + \alpha_{\eta^1 \sim \eta^2} = 1;\ \ \alpha_{\eta^2\eta^1} = 0$;

– incomparability: $\alpha_{\eta^1\eta^2} \neq 0,\ \ \alpha_{\eta^2\eta^1} \neq 0$.

The first two are common situations of full comparability. We are interested in the third situation. We shall proceed to identify the dominating option starting from iteration k + (ℵ – 1), the constraint being that for (k + ℵ) iteration the (k + ℵ) distribution results in the following relationship:

$$\alpha_{\eta^1\eta^2} + \alpha_{\eta^1 \sim \eta^2} \to 1;\ \ \alpha_{\eta^2\eta^1} \to 0.$$

Therefore, identifying the dominating option of a complex system follows the structural flowchart presented (fig. 3). It combines computational procedures to define the preference vector and the procedures of rational selection.

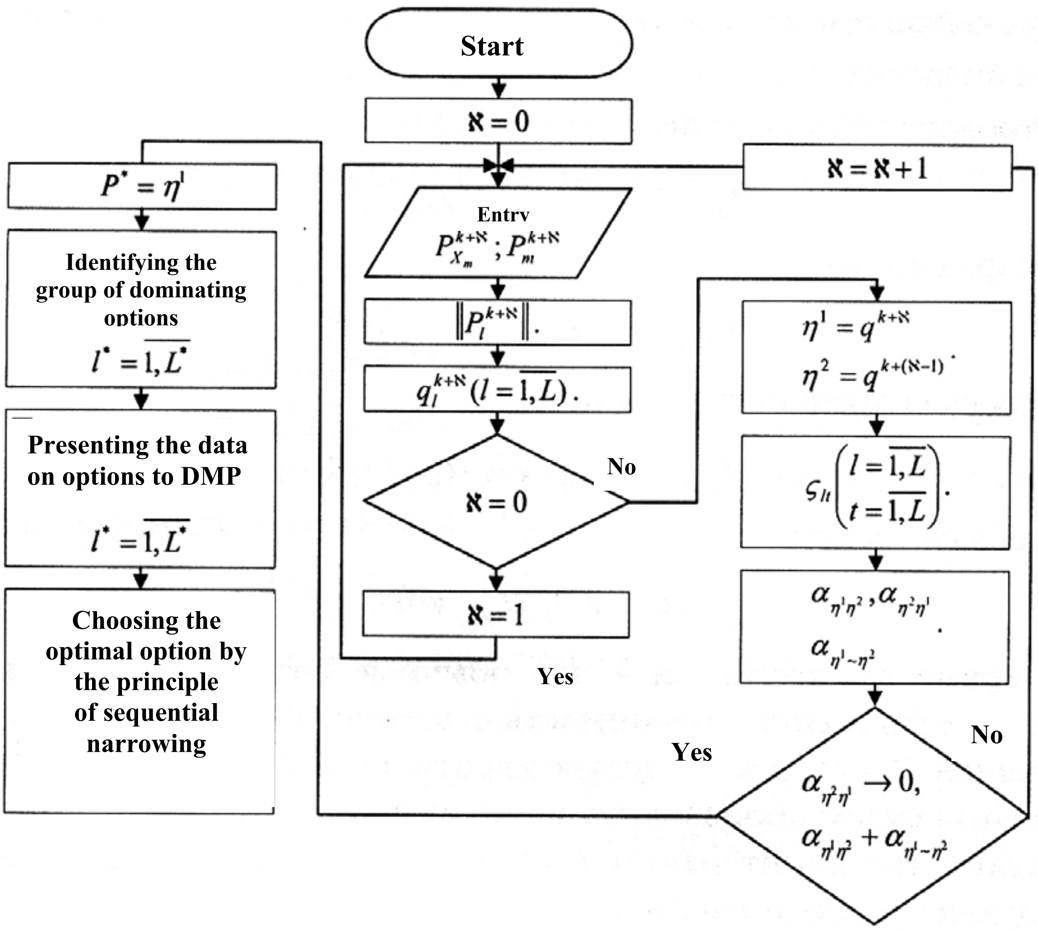

Figure 3. Structural flowchart of identifying the dominating option for a complex system

Conclusion

Development of a numeracy of dominant managerial decisions is possible on the basis of application of basic procedures of variation modeling and building of multialternative optimization models. At that an identification of the dominant option of a complex system is performed on the basis of the developed scheme, that combines procedures for determination of the preference vector and the rational choice procedures.

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Prof. Igor Lvovich, D. Sc.
Pan-European University, Bratislava, Slovakia
office@vivt.ru

Ing. Juraj Štefanovič, PhD.
Pan-European University, Bratislava, Slovakia
juraj.stefanovic@paneurouni.com